hep-th/0005086
ITP-UH-07/00

New N=2 supersymmetric gauge theories: The double tensor multiplet and its interactions

Friedemann Brandt

Institut für Theoretische Physik, Universität Hannover,
Appelstr. 2, D-30167 Hannover, Germany
E-mail: brandt@itp.uni-hannover.de

Abstract

The double tensor multiplet of D=4, N=2 supersymmetry, relevant to type IIB superstring vacua, is derived and its gauge invariant and N=2 supersymmetric interactions are analysed, both self-interactions and interactions with vector multiplets and hypermultiplets. Using deformation theory, it is shown that the lowest dimensional nontrivial interaction vertices of this type have dimension 5 and all dimension 5 vertices are determined. They give rise to new N=2 supersymmetric gauge theories of the 'exotic' type which are local but nonpolynomial in some of the fields and coupling constants. Explicit examples of such models are constructed.

PACS numbers: 11.30.Pb, 11.15.-q, 11.25.Mj
Keywords: extended supersymmetry, N=2 double tensor multiplet, new supersymmetric gauge theories

1 Introduction and summary

The paper is devoted to a special multiplet of four dimensional N=2 supersymmetry containing two real scalar fields, two 2-form gauge potentials and two Weyl fermions. This multiplet is particularly relevant to four dimensional type IIB superstring vacua with N=2 supersymmetry because there the dilaton is a member of such a multiplet, see e.g. [1]. The multiplet has been termed double tensor multiplet in the literature. However, to my knowledge, it has not been constructed explicitly up to know. This might have to do with the failure of an off-shell construction. In fact, one may be tempted to expect that the multiplet, with the above-mentioned field content, exists off-shell since the number of bosonic and fermionic degrees of freedom balance off-shell. However, as shown in section 2, such an off-shell realization of the N=2 supersymmetry algebra (possibly with a central charge, and modulo gauge transformations) is not compatible with the free action for the minimal field content.

Of course, this does not exclude that an off-shell formulation with additional (auxiliary) fields may exist but this question will not be addressed here. We shall thus work with the minimal field content and derive in section 2 the N=2 supersymmetry transformations for the free action. The N=2 supersymmetry algebra is realized on-shell modulo gauge transformations of the 2-form gauge potentials, without a central charge. The gauge transformations that occur in the algebra depend explicitly and at most linearly on the spacetime coordinates.[1] In terms of N=1 multiplets, the double tensor multiplet consists of two linear multiplets.

Having derived the free double tensor multiplet, we then study its interactions. Apart from self-interactions, the interactions with N=2 vector multiplets and hypermultiplets are analysed because of their relevance in the string theory context.[2] We impose that the action of the interacting theory be Poincaré invariant, gauge invariant and N=2 supersymmetric, but with possibly modified gauge and supersymmetry transformations as compared to the free theory. More precisely, we will study Poincaré invariant nontrivial continuous deformations of the free theory for one N=2 double tensor and an arbitrary number of vector multiplets and hypermultiplets. The analysis uses a systematic approach which is based on an expansion in the deformation parameters (coupling constants) and is briefly reviewed in section 3. The first order deformations must be nontrivial on-shell invariants of the free theory, i.e., field polynomials which are invariant on-shell (modulo total derivatives) under the gauge and supersymmetry transformations of the free theory.

These on-shell invariants can be determined at each mass dimension separately (assigning the standard dimensions to the fields). In section 4 all such invariants with dimensions ≤ 5 are determined. It turns out that there are no such invariants with dimensions ≤ 4; in particular, there are thus no nontrivial power counting renormalizable couplings of one double tensor multiplet to vector or hypermultiplets at all. All nontrivial couplings with dimension 5 are trilinear in the fields. There are three different

[1] The occurrence of gauge transformations involving explicitly the spacetime coordinates can be understood from a duality relation between the double tensor multiplet and the hypermultiplet. This will be explained in more detail and generality in a separate work.

[2] The interactions with the N=2 supergravity multiplet are not discussed here.

types of such couplings:

Type A: Self-couplings of the double tensor multiplet. They contain Freedman-Townsend interaction vertices [2].

Type B: Couplings of the double tensor multiplet to vector multiplets. These couplings are linear in the fields of the double tensor multiplet and in the fields of different vector multiplets; hence, couplings of this type involve at least two vector multiplets. They contain interaction vertices of the Henneaux-Knaepen type [3].

Type C: Couplings of the double tensor multiplet to hypermultiplets. These couplings are linear in the fields of the double tensor multiplet and quadratic in the fields of one or more hypermultiplets.

In section 5 we study whether these dimension 5 interaction vertices can be extended to higher orders in the deformation parameter(s). To that end we first reformulate the free double tensor multiplet by introducing an appropriate set of auxiliary fields. Then we point out a remarkable property common to all three types of first order deformations and discuss its consequences for the structure of the supersymmetry deformations. Finally we treat explicitly three examples, one for each coupling type described above.

The first example is the simplest one and arises from a type C coupling between the double tensor multiplet and one hypermultiplet. We complete this coupling to all orders in the deformation parameters. In this case the gauge transformations do not get deformed. In contrast, the supersymmetry transformations get deformed, without changing the supersymmetry algebra (the commutator of two deformed supersymmetry transformations is an ordinary translation plus a gauge transformation on-shell). The auxiliary fields mentioned above allow one to give the deformed action and supersymmetry transformations in a compact polynomial form. Upon elimination of the auxiliary fields, the deformed action and the supersymmetry transformations become non-polynomial in the deformation parameters and in the scalar fields of the hypermultiplet, but remain local (in fact, each term in the action is at most quadratic in derivatives of fields and the supersymmetry transformations of the elementary fields contain at most one derivative).

The second example arises from a coupling of type B between the double tensor multiplet and two vector multiplets. It is somewhat reminiscent of the N=2 supersymmetric gauge theories with the vector-tensor multiplet constructed in [4, 5] where the central charge of that multiplet was gauged (even though there is no central charge in the present case). In this case both the supersymmetry transformations and the gauge transformations get nontrivially deformed. Again, the supersymmetry algebra does not change: the commutator of two deformed supersymmetry transformations is a translation plus a deformed gauge transformation on-shell. As in the first example, the complete deformations of the action, gauge and supersymmetry transformations are given in a compact polynomial form using the auxiliary fields. Upon elimination of the auxiliary fields, the deformations of the action and symmetry transformations become non-polynomial in the deformation parameters and in the scalar and vector fields of the vector multiplets, but remain local.

As a third example we discuss the self-interactions of type A. Their completion to all orders is more involved and is not fully accomplished. However, the first order de-

formations of the gauge and supersymmetry transformations and the first and second order deformation of the action are computed explicitly in the formulation with the auxiliary fields. The result implies already that, in the formulation without auxiliary fields, the action and symmetry transformations would be non-polynomial in the deformation parameters and in the scalar fields and the 2-form gauge potentials. Furthermore it strongly suggests that all higher order deformations exist as well and allows one to guess the structure of the resulting full action and symmetry transformations.

Of course it should be stressed that these three examples are relatively simple N=2 supersymmetric gauge theories involving the double tensor multiplet. There may be more complicated models of this type. In particular, instead of discussing the couplings of type A, B or C separately, one may study linear combinations of them and investigate whether such linear combinations can be completed to higher orders. In fact, N=1 supersymmetric models of this more complicated type have been constructed in [6] and this suggests that analogous N=2 supersymmetric models exist as well. Section 6 comments on the use of the N=1 superfield construction in [6] in the present context. One may go even further and study whether couplings of type A, B or C can be combined with the well-known couplings relating only vector multiplets and hypermultiplets. Of course, there are many more open questions, such as the classification of first order interaction terms with dimensions ≥ 6, and the coupling of the double tensor multiplet to N=2 supergravity.

2 The free double tensor multiplet

General ansatz. The starting point is the standard free action for the above-described field content,

$$\int d^4x\,(\partial_\mu a^i\partial^\mu a^i - H^i_\mu H^{\mu i} - \mathrm{i}\chi\partial\bar{\chi} - \mathrm{i}\psi\partial\bar{\psi}) \tag{2.1}$$

where the a^i are the two real scalar fields ($i = 1, 2$), ψ and χ are the two 2-component Weyl fermions (their complex conjugates are denoted by $\bar{\psi}$ and $\bar{\chi}$), and H^i_μ are the Hodge-duals of the field strengths of the real 2-form gauge potentials $B^i_{\mu\nu}$,

$$H^{\mu i} = \tfrac{1}{2}\,\epsilon^{\mu\nu\rho\sigma}\partial_\nu B^i_{\rho\sigma}\ . \tag{2.2}$$

Furthermore, here and throughout the paper, summation over repeated indices of any kind (whether up or down) is understood (indices i are never lowered).[3]

To examine whether this free action is N=2 supersymmetric and to determine the supersymmetry transformations, we make the most general ansatz for linear N=2 supersymmetry transformations compatible with Poincaré covariance and with the dimensions of the fields. We write these transformations in the form

$$\delta^{(0)}_\xi = \xi^{\alpha i}D^{(0)i}_\alpha + \bar{\xi}^i_{\dot{\alpha}}\bar{D}^{(0)\dot{\alpha}i}\ . \tag{2.3}$$

[3]The remaining conventions and notation are analogous to those in [7], except that the Minkowski metric $\eta_{\mu\nu} = diag(1, -1, -1, -1)$ is used.

Here ξ^i_α are two constant anticommuting Weyl spinors (they are the parameters of the supersymmetry transformations), $D^{(0)i}_\alpha$ are the generators of the corresponding supersymmetry transformations and $\bar\xi^i_{\dot\alpha}$ and $\bar D^{(0)i}_{\dot\alpha}$ are the complex conjugates of ξ^i_α and $D^{(0)i}_\alpha$ respectively. The ansatz takes then the form

$$\begin{aligned}
D^{(0)i}_\alpha a^j &= \tfrac{1}{2}\,(M^{ij}\chi_\alpha + N^{ij}\psi_\alpha)\\
D^{(0)i}_\alpha B^j_{\mu\nu} &= R^{ij}(\sigma_{\mu\nu}\chi)_\alpha + S^{ij}(\sigma_{\mu\nu}\psi)_\alpha\\
D^{(0)i}_\alpha \chi_\beta &= 0\\
D^{(0)i}_\alpha \psi_\beta &= 0\\
D^{(0)i}_\alpha \bar\chi_{\dot\alpha} &= X^{ij}\partial_{\alpha\dot\alpha}a^j + Y^{ij}H^j_{\alpha\dot\alpha} + Z^{ij}\sigma^\mu_{\alpha\dot\alpha}\partial^\nu B^j_{\nu\mu}\\
D^{(0)i}_\alpha \bar\psi_{\dot\alpha} &= \hat X^{ij}\partial_{\alpha\dot\alpha}a^j + \hat Y^{ij}H^j_{\alpha\dot\alpha} + \hat Z^{ij}\sigma^\mu_{\alpha\dot\alpha}\partial^\nu B^j_{\nu\mu}
\end{aligned}\tag{2.4}$$

where the coefficients M^{ij}, ..., $\hat Z^{ij}$ are complex numbers which are to be determined from the following requirements: (i) the free Lagrangian must be invariant modulo a total derivative under the above transformations, for any choice of ξ^i_α; (ii) the commutators of these transformations must fulfill the N=2 supersymmetry algebra at least on-shell, modulo gauge transformations and possibly with some central charges. Clearly, if there is a set of coefficients M^{ij}, ..., $\hat Z^{ij}$ which fulfills these requirements, it cannot be unique because the Lagrangian is also invariant under separate $SO(2)$ transformations of the a^i and $B^i_{\mu\nu}$ and under $SU(2)$ and $U(1)$ transformations of the fermions. Hence, there is some freedom in writing the supersymmetry transformations owing to these global symmetries.

(i) is equivalent to the requirement that all $D^{(0)i}_\alpha$-transformations of the free Lagrangian be total derivatives (the $\bar D^{(0)i}_{\dot\alpha}$-transformations do not give additional conditions since the free Lagrangian is real modulo a total derivative). It is straightforward to verify that this imposes precisely the following conditions:

$$\mathrm{i}X^{ij} = M^{ij},\quad \mathrm{i}\hat X^{ij} = N^{ij},\quad Y^{ij} = -R^{ij},\quad \hat Y^{ij} = -S^{ij},\quad Z^{ij} = 0 = \hat Z^{ij}. \tag{2.5}$$

Using this, the ansatz reduces to

$$\begin{aligned}
D^{(0)i}_\alpha a^j &= \tfrac{1}{2}\,(M^{ij}\chi_\alpha + N^{ij}\psi_\alpha)\\
D^{(0)i}_\alpha B^j_{\mu\nu} &= R^{ij}(\sigma_{\mu\nu}\chi)_\alpha + S^{ij}(\sigma_{\mu\nu}\psi)_\alpha\\
D^{(0)i}_\alpha \chi_\beta &= 0\\
D^{(0)i}_\alpha \psi_\beta &= 0\\
D^{(0)i}_\alpha \bar\chi_{\dot\alpha} &= -\mathrm{i}M^{ij}\partial_{\alpha\dot\alpha}a^j - R^{ij}H^j_{\alpha\dot\alpha}\\
D^{(0)i}_\alpha \bar\psi_{\dot\alpha} &= -\mathrm{i}N^{ij}\partial_{\alpha\dot\alpha}a^j - S^{ij}H^j_{\alpha\dot\alpha}\ .
\end{aligned}\tag{2.6}$$

The remaining coefficients are subject to requirement (ii).

Absence of an off-shell representation. There are no coefficients M^{ij}, N^{ij}, R^{ij}, S^{ij} such that the transformations (2.6) give an off-shell representation of the standard

N=2 supersymmetry algebra (modulo gauge transformations and possibly with a central charge). Such an off-shell representation would require in particular $\{D_\alpha^{(0)i},\bar D_{\dot\alpha}^{(0)j}\}=-\mathrm{i}\,\delta^{ij}\partial_{\alpha\dot\alpha}$ on the fermions. However, this leads to inconsistent equations for the coefficients M^{ij}, N^{ij}, R^{ij}, S^{ij}. Indeed, one finds

$$\begin{array}{l}\{D_\alpha^{(0)i},\bar D_{\dot\alpha}^{(0)j}\}\chi_\beta=-\mathrm{i}\,\delta^{ij}\partial_{\alpha\dot\alpha}\chi_\beta\ ,\ \{D_\alpha^{(0)i},\bar D_{\dot\alpha}^{(0)j}\}\psi_\beta=-\mathrm{i}\,\delta^{ij}\partial_{\alpha\dot\alpha}\psi_\beta\\ \Leftrightarrow\quad MM^\dagger=NN^\dagger=RR^\dagger=SS^\dagger=1,\quad MN^\dagger=RS^\dagger=0\end{array}$$

where M, N, R, S are the matrices with entries M^{ij}, N^{ij}, R^{ij}, S^{ij} respectively.

Determination of the supersymmetry transformations. Coefficients M^{ij}, N^{ij}, R^{ij}, S^{ij} which yield an on-shell representation of the N=2 supersymmetry algebra can be found by dualizing two scalar fields of an N=2 hypermultiplet. A hypermultiplet contains two complex scalar fields φ^i ($i=1,2$) and two Weyl fermions (see section 4 for details). One may decompose the scalar fields into real and imaginary parts, $\varphi^i=a^i+\mathrm{i}b^i$, and then "dualize" the imaginary parts according to $\partial_\mu b^i\to\epsilon^{ji}H_\mu^j$ (the use of ϵ^{ji} is a pure convention; other choices are available owing to the above-mentioned freedom in writing the supersymmetry transformations). The transformations of the $B_{\mu\nu}^i$ are chosen such that the transformations of $\partial_\mu b^i$ and $\epsilon^{ji}H_\mu^j$ coincide on-shell, i.e., when the free field equations for the fermions are used. This dualization procedure yields automatically transformations that fulfill on-shell the N=2 supersymmetry algebra modulo gauge transformations. Furthermore the resulting transformations are indeed of the form (2.6). Hence, they are the sought N=2 supersymmetry transformations for the double tensor multiplet. One finds

$$\begin{array}{rcl}D_\alpha^{(0)i}a^j&=&\frac12(\delta^{ij}\chi_\alpha+\epsilon^{ij}\psi_\alpha)\\ D_\alpha^{(0)i}B_{\mu\nu}^j&=&\epsilon^{ij}(\sigma_{\mu\nu}\chi)_\alpha+\delta^{ij}(\sigma_{\mu\nu}\psi)_\alpha\\ D_\alpha^{(0)i}\chi_\beta&=&0\\ D_\alpha^{(0)i}\psi_\beta&=&0\\ D_\alpha^{(0)i}\bar\chi_{\dot\alpha}&=&-\mathrm{i}\,\partial_{\alpha\dot\alpha}a^i-\epsilon^{ij}H_{\alpha\dot\alpha}^j\\ D_\alpha^{(0)i}\bar\psi_{\dot\alpha}&=&-\mathrm{i}\,\epsilon^{ij}\partial_{\alpha\dot\alpha}a^j-H_{\alpha\dot\alpha}^i\ .\end{array}\tag{2.7}$$

From these transformations one reads off that the double tensor multiplet consists of two N=1 linear multiplets. For instance, the N=1 linear multiplets with respect to $D_\alpha^{(0)1}$ are $(a^1,B_{\mu\nu}^2,\chi)$ and $(a^2,B_{\mu\nu}^1,\psi)$.

N=2 supersymmetry algebra. As remarked above, the transformations (2.7) yield an on-shell representation of the N=2 supersymmetry algebra because they can be obtained by dualizing two scalar fields of a hypermultiplet. One finds that the algebra is realized on-shell modulo gauge transformations of the $B_{\mu\nu}^i$, without a central charge. More precisely, the algebra is realized even off-shell on the scalar fields,

$$\begin{array}{rcl}\{D_\alpha^{(0)i},\bar D_{\dot\alpha}^{(0)j}\}a^k&=&-\mathrm{i}\,\delta^{ij}\partial_{\alpha\dot\alpha}a^k\\ \{D_\alpha^{(0)i},D_\beta^{(0)j}\}a^k&=&0\end{array}\tag{2.8}$$

while it holds on-shell on the fermions,

$$\begin{array}{rcl}
\{D^{(0)i}_\alpha,\bar D^{(0)j}_{\dot\alpha}\}\chi_\beta &=& -\mathrm{i}\,\delta^{ij}\partial_{\alpha\dot\alpha}\chi_\beta-\mathrm{i}\,\epsilon^{ij}\epsilon_{\alpha\beta}\partial_{\gamma\dot\alpha}\psi^\gamma\approx-\mathrm{i}\,\delta^{ij}\partial_{\alpha\dot\alpha}\chi_\beta\\
\{D^{(0)i}_\alpha,D^{(0)j}_\beta\}\chi_\gamma &=& 0\\
\{D^{(0)i}_\alpha,D^{(0)j}_\beta\}\bar\chi_{\dot\alpha} &=& \mathrm{i}\,\epsilon^{ij}\epsilon_{\alpha\beta}\partial_{\gamma\dot\alpha}\psi^\gamma\approx 0\\
\{D^{(0)i}_\alpha,\bar D^{(0)j}_{\dot\alpha}\}\psi_\beta &=& -\mathrm{i}\,\delta^{ij}\partial_{\alpha\dot\alpha}\psi_\beta+\mathrm{i}\,\epsilon^{ij}\epsilon_{\alpha\beta}\partial_{\gamma\dot\alpha}\chi^\gamma\approx-\mathrm{i}\,\delta^{ij}\partial_{\alpha\dot\alpha}\psi_\beta\\
\{D^{(0)i}_\alpha,D^{(0)j}_\beta\}\psi_\gamma &=& 0\\
\{D^{(0)i}_\alpha,D^{(0)j}_\beta\}\bar\psi_{\dot\alpha} &=& -\mathrm{i}\,\epsilon^{ij}\epsilon_{\alpha\beta}\partial_{\gamma\dot\alpha}\chi^\gamma\approx 0
\end{array}\qquad(2.9)$$

where $\approx$ denotes on-shell equality. Finally, on the 2-form gauge potentials one has

$$\begin{array}{rcl}
\{D^{(0)i}_\alpha,\bar D^{(0)j}_{\dot\alpha}\}B^k_{\mu\nu} &=& -\mathrm{i}\,\delta^{ij}\partial_{\alpha\dot\alpha}B^k_{\mu\nu}+2\partial_{[\mu}\Lambda^{ijk}_{\nu]\alpha\dot\alpha}-2\epsilon^{ij}\epsilon^{kl}x_{\alpha\dot\alpha}\partial_{[\mu}H^l_{\nu]}\\
&\approx& -\mathrm{i}\,\delta^{ij}\partial_{\alpha\dot\alpha}B^k_{\mu\nu}+\partial_\mu\Lambda^{ijk}_{\nu\alpha\dot\alpha}-\partial_\nu\Lambda^{ijk}_{\mu\alpha\dot\alpha}\\
\{D^{(0)i}_\alpha,D^{(0)j}_\beta\}B^k_{\mu\nu} &=& 0
\end{array}\qquad(2.10)$$

where

$$\Lambda^{ijk}_{\mu\nu}=\tfrac{\mathrm{i}}{2}\,\eta_{\mu\nu}(\delta^{jk}\epsilon^{il}+\delta^{il}\epsilon^{jk}+\delta^{ik}\epsilon^{jl}+\delta^{jl}\epsilon^{ik})a^l+\epsilon^{ij}\epsilon^{kl}x_\nu H^l_\mu-\mathrm{i}\,\delta^{ij}B^k_{\mu\nu}\ .\qquad(2.11)$$

Note that the terms with the $\Lambda^{ijk}_{\mu\nu}$ in (2.10) are special gauge transformations. Hence, the N=2 supersymmetry algebra is indeed represented on the 2-form gauge potentials on-shell modulo gauge transformations with "gauge parameters" involving the $\Lambda^{ijk}_{\mu\nu}$.

To summarize, in the notation (2.3) one has on all fields

$$[\delta^{(0)}_\xi,\delta^{(0)}_{\xi'}]\approx\xi^\mu\partial_\mu+\delta^{(0)}_\Lambda\qquad(2.12)$$

where ξ^μ is a constant vector involving the supersymmetry parameters,

$$\xi^\mu=\mathrm{i}\,(\xi^{i\prime}\sigma^\mu\bar\xi^i-\xi^i\sigma^\mu\bar\xi^{i\prime}),\qquad(2.13)$$

and $\delta^{(0)}_\Lambda$ is a special gauge transformation,

$$\delta^{(0)}_\Lambda B^i_{\mu\nu}=\partial_\mu\Lambda^i_\nu-\partial_\nu\Lambda^i_\mu\ ,\quad \Lambda^i_\mu=\Lambda^{jki}_{\mu\nu}\,(\xi^j\sigma^\nu\bar\xi^{k\prime}-\xi^{j\prime}\sigma^\nu\bar\xi^k).\qquad(2.14)$$

Note that both these gauge transformations and the terms with equations of motion which appear in the commutator on $B^i_{\mu\nu}$ involve explicitly the spacetime coordinates x^μ. It is easy to check that the sum of these terms actually does not depend explicitly on the x^μ (it cannot because there is no explicit x-dependence in the supersymmetry transformations): the relevant terms in the first anticommutator (2.10) combine to the x-independent term $2\epsilon^{ij}\epsilon^{kl}H^l_{[\nu}\sigma_{\mu]\alpha\dot\alpha}$. However, this term is not to be interpreted as some kind of "central charge" in the supersymmetry algebra because it equals a gauge transformation on-shell. The point is that this gauge transformation depends explicitly on the x^μ, owing to

$$H^l_{[\nu}\sigma_{\mu]\alpha\dot\alpha}=\partial_{[\mu}(H^l_{\nu]}x_{\alpha\dot\alpha})-x_{\alpha\dot\alpha}\partial_{[\mu}H^l_{\nu]}\approx\partial_{[\mu}(H^l_{\nu]}x_{\alpha\dot\alpha}).$$

A more complete discussion of the rôle and origin of such terms in supersymmetry algebras will be given elsewhere.

3 Brief description of deformation theory

To study interactions involving the double tensor multiplet, we shall start from the free action for one double tensor and an arbitrary number of vector multiplets and hypermultiplets. We shall seek deformations of the free action which are invariant under the standard Poincaré transformations and under possibly deformed gauge and supersymmetry transformations. The requirement that these deformations be continuous means that the deformed Lagrangian L, the corresponding gauge transformations δ_ε and supersymmetry transformations δ_ξ can be expanded in deformation parameters (coupling constants) according to

$$\begin{aligned} L &= L^{(0)} + L^{(1)} + L^{(2)} + \dots && (3.1)\\ \delta_\varepsilon &= \delta_\varepsilon^{(0)} + \delta_\varepsilon^{(1)} + \delta_\varepsilon^{(2)} + \dots && (3.2)\\ \delta_\xi &= \delta_\xi^{(0)} + \delta_\xi^{(1)} + \delta_\xi^{(2)} + \dots && (3.3)\\ \delta_\xi^{(k)} &= \xi^{\alpha i} D_\alpha^{(k)i} + \bar\xi^i_{\dot\alpha} \bar D^{(k)\dot\alpha i} && (3.4) \end{aligned}$$

where $L^{(k)}$, $\delta_\varepsilon^{(k)}$, $\delta_\xi^{(k)}$, $D_\alpha^{(k)i}$ and $\bar D^{(k)\dot\alpha i}$ have order k in the deformation parameters, and $L^{(0)}$, $\delta_\varepsilon^{(0)}$ and $\delta_\xi^{(0)}$ denote the free Lagrangian, its gauge symmetries and supersymmetry transformations respectively. ε and ξ denote collectively the "parameters" of gauge and N=2 supersymmetry transformations respectively, i.e., the ε's are arbitrary fields whereas the ξ's are constant anticommuting spinors.

The invariance requirements in the deformed theory are

$$\delta_\varepsilon L \simeq 0, \quad \delta_\xi L \simeq 0, \tag{3.5}$$

where $\simeq$ is equality modulo total derivatives. The analysis can now be performed "perturbatively" by expanding these invariance requirements in the deformation parameters. At first order, this requires

$$\begin{aligned} &\delta_\varepsilon^{(0)} L^{(1)} + \delta_\varepsilon^{(1)} L^{(0)} \simeq 0 && (3.6)\\ &\delta_\xi^{(0)} L^{(1)} + \delta_\xi^{(1)} L^{(0)} \simeq 0. && (3.7) \end{aligned}$$

These equations can also be cast in the form

$$\delta_\varepsilon^{(0)} L^{(1)} + \sum_\Phi (\delta_\varepsilon^{(1)} \Phi) \frac{\delta L^{(0)}}{\delta \Phi} \simeq 0 \tag{3.8}$$

$$\delta_\xi^{(0)} L^{(1)} + \sum_\Phi (\delta_\xi^{(1)} \Phi) \frac{\delta L^{(0)}}{\delta \Phi} \simeq 0 \tag{3.9}$$

where the sum $\sum_\Phi$ runs over all fields and $\delta L^{(0)}/\delta\Phi$ is the Euler-Lagrange derivative of the free Lagrangian with respect to Φ. This shows that $L^{(1)}$ has to be invariant on-shell under the zeroth order transformations $\delta_\varepsilon^{(0)}$ and $\delta_\xi^{(0)}$, where this on-shell invariance refers to the free field equations $\delta L^{(0)}/\delta\Phi = 0$. Furthermore, we are only interested in nontrivial deformations, i.e. in deformations that cannot be removed through mere

field redefinitions. The free Lagrangian changes under infinitesimal field redefinitions $\Delta\Phi$ through terms $\sum_\Phi(\Delta\Phi)\delta L^{(0)}/\delta\Phi + \partial_\mu M^\mu$. These are terms which vanish on-shell in the free theory modulo total derivatives. Terms of this form are thus trivial and can be neglected without loss of generality.

Hence, the first step of the perturbative approach to the deformation problem is the determination of nontrivial on-shell invariants of the free theory. The first order deformation of the free Lagrangian is a linear combination of these on-shell invariants. The corresponding first order deformations of the gauge and supersymmetry transformations are the coefficient functions of the Euler-Lagrange derivatives $\delta L^{(0)}/\delta\Phi$ which appear in (3.8) and (3.9).

At second order, (3.5) gives

$$\delta_\varepsilon^{(0)} L^{(2)} + \delta_\varepsilon^{(1)} L^{(1)} + \sum_\Phi (\delta_\varepsilon^{(2)}\Phi)\,\frac{\delta L^{(0)}}{\delta\Phi} \simeq 0 \tag{3.10}$$

$$\delta_\xi^{(0)} L^{(2)} + \delta_\xi^{(1)} L^{(1)} + \sum_\Phi (\delta_\xi^{(2)}\Phi)\,\frac{\delta L^{(0)}}{\delta\Phi} \simeq 0. \tag{3.11}$$

These equations require that $\delta_\varepsilon^{(1)} L^{(1)}$ and $\delta_\xi^{(1)} L^{(1)}$ be in the image of $\delta_\varepsilon^{(0)}$ and $\delta_\xi^{(0)}$ respectively, at least on-shell (with respect to the free theory) and modulo total derivatives. This can impose relations between the coefficients of the on-shell invariants in $L^{(1)}$ (it can even set some of these coefficients to zero). Additional relations between these coefficients can be imposed by the equations arising from (3.5) at even higher orders of the deformation problem.

Such relations between the coefficients in $L^{(1)}$ have a cohomological characterization. In fact, the whole deformation theory sketched above can be usefully reformulated as a cohomological problem in the framework of an extended BRST formalism. The relevant cohomology is that of an extended BRST differential $s^{(0)}$ which encodes the global supersymmetry transformations, the gauge transformations and the equations of motion of the free theory. This cohomological formulation of the deformation theory is described in [8] and extends the deformation theory developed in [9]. In the cohomological approach, the classification of the on-shell invariants of the free theory amounts to compute the cohomology of the extended BRST differential of the free theory in the space of local functionals with ghost number 0. The deformation problem at orders ≥ 2 is controlled by the same cohomology, but now at ghost number 1: the cohomology classes at ghost number 1 give the possible obstructions to the existence of a deformation at orders ≥ 2. This rôle of the cohomology at ghost number 1 is similar to the characterization of candidate anomalies through the BRST cohomology in the quantum field theoretical context.

4 First order deformations of dimension ≤ 5

4.1 Free action

The input for the deformation theory is the free action for one double tensor multiplet and a set of vector multiplets and hypermultiplets.[4] The fields of a vector multiplet are a real gauge field A_μ, a complex scalar field ϕ and two Weyl fermions λ^i_α. The complex conjugates of ϕ and λ^i_α are denoted by $\bar\phi$ and $\bar\lambda^i_{\dot\alpha}$ respectively (note: as before, complex conjugation does not lower the index i). The fields of a hypermultiplet are two complex scalar fields φ^i and two Weyl fermions ρ_α and η_α. Their complex conjugates are denoted by $\bar\varphi^i$, $\bar\rho_{\dot\alpha}$ and $\bar\eta_{\dot\alpha}$. The free Lagrangian is

$$\begin{aligned} L^{(0)} &= \partial_\mu a^i\partial^\mu a^i - H^i_\mu H^{\mu i} - \mathrm{i}\chi\partial\bar\chi - \mathrm{i}\psi\partial\bar\psi \\ &\quad -\tfrac{1}{4}\,F^A_{\mu\nu}F^{\mu\nu A} + \tfrac{1}{2}\,\partial_\mu\phi^A\partial^\mu\bar\phi^A - 2\mathrm{i}\lambda^{iA}\partial\bar\lambda^{iA} \\ &\quad +\partial_\mu\varphi^{ia}\partial^\mu\bar\varphi^{ia} - \mathrm{i}\rho^a\partial\bar\rho^a - \mathrm{i}\eta^a\partial\bar\eta^a \end{aligned} \tag{4.1}$$

where A and a label the vector multiplets and hypermultiplets respectively and $F^A_{\mu\nu}$ is the field strength of A^A_μ,

$$F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu\ .$$

Using again the notation (2.3), the N=2 supersymmetry transformations of the vector multiplets and hypermultiplets read

$$\begin{aligned} D^{(0)i}_\alpha A^A_\mu &= \epsilon^{ij}(\sigma_\mu\bar\lambda^{jA})_\alpha \\ D^{(0)i}_\alpha \phi^A &= 2\lambda^{iA}_\alpha \\ D^{(0)i}_\alpha \bar\phi^A &= 0 \\ D^{(0)i}_\alpha \lambda^{jA}_\beta &= -\tfrac{\mathrm{i}}{2}\,\epsilon^{ij}\sigma^{\mu\nu}_{\alpha\beta}F^A_{\mu\nu} \\ D^{(0)i}_\alpha \bar\lambda^{jA}_{\dot\alpha} &= -\tfrac{\mathrm{i}}{2}\,\delta^{ij}\,\partial_{\alpha\dot\alpha}\bar\phi^A \\ D^{(0)i}_\alpha \varphi^{ja} &= \epsilon^{ij}\rho^a_\alpha \\ D^{(0)i}_\alpha \bar\varphi^{ja} &= \delta^{ij}\eta^a_\alpha \\ D^{(0)i}_\alpha \rho^a_\beta &= 0 \\ D^{(0)i}_\alpha \eta^a_\beta &= 0 \\ D^{(0)i}_\alpha \bar\rho^a_{\dot\alpha} &= -\mathrm{i}\,\epsilon^{ij}\partial_{\alpha\dot\alpha}\bar\varphi^{ja} \\ D^{(0)i}_\alpha \bar\eta^a_{\dot\alpha} &= -\mathrm{i}\,\partial_{\alpha\dot\alpha}\varphi^{ia} \end{aligned} \tag{4.2}$$

The gauge symmetries act nontrivially only on A^A_μ and $B^i_{\mu\nu}$ according to

$$\delta_\varepsilon{}^{(0)}A^A_\mu = \partial_\mu\varepsilon^A \tag{4.3}$$

$$\delta_\varepsilon{}^{(0)}B^i_{\mu\nu} = \partial_\mu\varepsilon^i_\nu - \partial_\nu\varepsilon^i_\mu\ . \tag{4.4}$$

4.2 General remarks and strategy

As described in the previous section, the first step within the deformation approach is the determination of the nontrivial on-shell invariants of the free theory. The classification of

[4] The generalization to the case with more than one double tensor multiplet is left to the reader.

these on-shell invariants can be carried out separately in subspaces of field polynomials with definite dimension and degree in the fields of the various multiplets.

To specify these subspaces, we assign dimension 1 to all bosons (scalar fields, vector fields, 2-form gauge potentials) and to the spacetime derivatives ∂_μ, dimension 3/2 to all fermions, and dimension 0 to the gauge parameters ε^A and ε^i_μ. Then the supersymmetry transformations $D^{(0)i}_\alpha$ have dimension 1/2 and the gauge transformations $\delta_\varepsilon{}^{(0)}$ have dimension 0. Furthermore the $D^{(0)i}_\alpha$ are linear in the fields and do not mix fields of different supermultiplets. One can therefore classify the on-shell invariants separately in subspaces of field polynomials characterized by (d, N_{TT}, N_V, N_H) where d is the dimension, and N_{TT}, N_V and N_H are the degrees in the fields of the double tensor multiplet, vector multiplets and hyper multiplets respectively.

To specify the field polynomials in a given subspace, it is helpful to denote by N_b, N_f and N_∂ the degree in the bosons, fermions and spacetime derivatives respectively. A field polynomial with a definite degree N_Φ in all fields and a definite dimension d fulfills thus

$$N_b + N_f = N_\Phi, \quad N_b + N_\partial + \tfrac{3}{2}\, N_f = d. \tag{4.5}$$

This yields in particular

$$N_\partial + \tfrac{1}{2}\, N_f = d - N_\Phi. \tag{4.6}$$

A field polynomial characterized by (d, N_{TT}, N_V, N_H) has $N_\Phi = N_{TT}+N_V+N_H$ and thus can contain only terms with $(N_\partial, N_f) = (d - N_\Phi, 0)$, $(d - N_\Phi - 1, 2)$, ..., $(0, 2d - 2N_\Phi)$.

Note that N_Φ ranges from 1 to d (for given value of d). It is easy to verify that the values $N_\Phi = 1$ and $N_\Phi = d$ do not give nontrivial on-shell invariants because we impose also Poincaré invariance. Indeed, in both cases the only Poincaré invariant field polynomials which are nontrivial and on-shell gauge invariant modulo a total derivative are polynomials in the undifferentiated scalar fields (there are gauge invariants with $N_\Phi = 1$, such as each A^A_μ, but they are not Poincaré invariant); no such polynomial is on-shell supersymmetric modulo a total derivative. In particular, there are no on-shell invariants with $d = 2$. In the following we shall discuss the remaining cases with $d = 3, 4, 5$ and $N_{TT} \geq 1$. The discussion covers both true interaction terms and terms quadratic in the fields.

The computations have been done in two steps: 1. Determination of the most general nontrivial real Poincaré invariant field polynomial $P_{(d,N_{TT},N_V,N_H)}$ which satisfies $\delta^{(0)}_\varepsilon P_{(d,N_{TT},N_V,N_H)} \sim 0$ where $\sim$ denotes on-shell equality in the free theory modulo a total derivative; 2. Imposing $D^{(0)i}_\alpha P_{(d,N_{TT},N_V,N_H)} \sim 0$ which yields then the first order deformations $L^{(1)}_{(d,N_{TT},N_V,N_H)}$ for each case (d, N_{TT}, N_V, N_H) separately. Of course, $P_{(d,N_{TT},N_V,N_H)}$ and $L^{(1)}_{(d,N_{TT},N_V,N_H)}$ are determined only modulo trivial terms which do not matter. We will give "minimal expressions" for these polynomials containing as many strictly gauge invariant terms as possible, and as few trivial terms as possible.

4.3 d=3

The polynomials to be discussed are those with $(d, N_{TT}, N_V, N_H) = (3, 2, 0, 0)$, $(3, 1, 0, 1)$ and $(3, 1, 1, 0)$ respectively. The only Poincaré invariant terms $(d, N_{TT}, N_V, N_H) =$

$(3,2,0,0)$ are bilinears in the fermions of the double tensor multiplet, $P_{(3,2,0,0)} = k_1\bar\chi\bar\chi + k_2\bar\psi\bar\psi + k_3\bar\chi\bar\psi + \text{c.c.}$ where $k_1, k_2, k_3 \in \mathbb{C}$. It is straightforward to verify that no such term is on-shell supersymmetric modulo a total derivative (unless $k_1 = k_2 = k_3 = 0$) because the H^i_μ occur in the transformation of $\bar\psi$ and $\bar\chi$.

Similarly, the only Poincaré invariant terms with $(d, N_{TT}, N_V, N_H) = (3,1,0,1)$ are linear combinations of bilinears in fermions, with one fermion of the double tensor multiplet and one fermion of a hypermultiplet respectively. Again, no nonvanishing linear combination of this type is on-shell supersymmetric modulo a total derivative owing to the presence of the H^i_μ in the transformations of $\bar\psi$ and $\bar\chi$.

The case $(d, N_{TT}, N_V, N_H) = (3,1,1,0)$ is more involved because now there are both bilinears in the fermions and terms with bosons which are Poincaré invariant and on-shell gauge invariant modulo a total derivative,

$$P_{(3,1,1,0)} = k^i_1 A_\mu \partial^\mu a^i + k^i_2 A_\mu H^{\mu i} + k^i_3 \lambda^i \psi + k^i_4 \lambda^i \chi + \bar k^i_3 \bar\lambda^i \bar\psi + \bar k^i_4 \bar\lambda^i \bar\chi$$

where $k^i_1, k^i_2 \in \mathbb{R}$, $k^i_3, k^i_4 \in \mathbb{C}$ (since the supersymmetry transformations do not mix the fields of different vector multiplets, the discussion can be made for each vector multiplet separately and we have dropped the index A). Modulo trivial terms, $D^{(0)i}_\alpha P_{(3,1,1,0)}$ contains terms proportional to $F_{\mu\nu}\sigma^{\mu\nu}\psi$, $F_{\mu\nu}\sigma^{\mu\nu}\chi$ and $H^j_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha k}$. $D^{(0)i}_\alpha P_{(3,1,1,0)} \sim 0$ requires that the coefficients of these terms vanish. This imposes

$$\begin{aligned}
\epsilon^{ij} k^j_1 - \mathrm{i} k^i_2 + \mathrm{i}\epsilon^{ij} k^j_3 &= 0 \\
k^i_1 - \mathrm{i}\epsilon^{ij} k^j_2 + \mathrm{i}\epsilon^{ij} k^j_4 &= 0 \\
\epsilon^{ik} k^j_2 - \delta^{ij} \bar k^k_3 - \epsilon^{ij} \bar k^k_4 &= 0
\end{aligned}$$

which yield

$$k^i_1 = k^i_2 = k^i_3 = k^i_4 = 0.$$

Remark. Recall that the cases $N_{TT} = 0$ are not discussed here. In fact, there are nontrivial on-shell invariants with $d = 3$ and $N_{TT} = 0$. These are the mass terms for the fermions of the hypermultiplets which have $(d, N_{TT}, N_V, N_H) = (3,0,0,2)$. As the above discussion shows, these mass terms have no counterparts with $N_{TT} > 0$ owing to the presence of the 2-form gauge potentials in the double tensor multiplet (more precisely: the presence of H^i_μ in the supersymmetry transformations of $\bar\psi$ and $\bar\chi$).

4.4 d=4

We need to discuss the cases $N_\Phi = 2$ and $N_\Phi = 3$. The cases $N_\Phi = 2$ are easy: there are simply no nontrivial terms which are quadratic in the fields, Poincaré invariant and on-shell gauge invariant modulo a total derivative. Indeed, the candidate terms $(d, N_{TT}, N_V, N_H) = (4,2,0,0)$ are linear combinations of the terms $H^i_\mu H^{\mu j}$, $H^i_\mu \partial^\mu a^j$, $\partial_\mu a^i \partial^\mu a^j$, $\psi\partial\bar\psi$, $\chi\partial\bar\chi$, $\chi\partial\bar\psi$ (modulo trivial ones); candidate terms $(d, N_{TT}, N_V, N_H) = (4,1,1,0)$ or $(4,1,0,1)$ are linear combinations of terms such as $\partial_\mu a^i \partial^\mu \phi^A$, $H^i_\mu \partial^\mu \varphi^a$, $\lambda^{iA}\partial\bar\psi$ etc.; all these terms vanish on-shell (in the free theory) modulo a total derivative. Note that the free Lagrangian itself is of this type and clearly vanishes on-shell modulo a total derivative.

The various terms with $N_\Phi = 3$ and $N_{TT} \geq 1$ are $(d, N_{TT}, N_V, N_H) = (4,3,0,0)$, $(4,2,1,0)$, $(4,2,0,1)$, $(4,1,2,0)$, $(4,1,1,1)$, $(4,1,0,2)$. Owing to (4.6), the field polynomials in these subsectors contain only terms with $(N_\partial, N_f) = (1,0)$ or $(0,2)$.

Consider first the case $(d, N_{TT}, N_V, N_H) = (4,3,0,0)$. Poincaré invariance excludes all terms with $(N_\partial, N_f) = (1,0)$. The most general Poincaré invariant field polynomial with $(N_\partial, N_f) = (0,2)$ is $k_1^i a^i \bar\psi\bar\psi + k_2^i a^i \bar\psi\bar\chi + k_3^i a^i \bar\chi\bar\chi + k_4^i B^i_{\mu\nu}\bar\psi\bar\sigma^{\mu\nu}\bar\chi +$c.c. The requirement that it be gauge invariant on-shell modulo a total derivative yields $k_4^i = 0$, i.e., $P_{(4,3,0,0)} = k_1^i a^i \bar\psi\bar\psi + k_2^i a^i \bar\psi\bar\chi + k_3^i a^i \bar\chi\bar\chi +$ c.c.. $D_\alpha^{(0)i} P_{(4,3,0,0)} \sim 0$ imposes $k_1^i = k_2^i = k_3^i = 0$.

An analogous discussion shows that the other cases with $N_V = 0$ $[(d, N_{TT}, N_V, N_H) = (4,2,0,1)$ and $(4,1,0,2)]$ do not yield nontrivial on-shell invariants.

The remaining cases $(d, N_{TT}, N_V, N_H) = (4,2,1,0)$, $(4,1,2,0)$ and $(4,1,1,1)$ are more involved because now there are terms with $(N_\partial, N_f) = (1,0)$ and a new type of terms with $(N_\partial, N_f) = (0,2)$ which are Poincaré invariant and gauge invariant on-shell modulo total derivatives. These terms are of the type $A^A_\mu j^\mu_A$ where j^μ_A are Noether currents of the free theory which have dimension 3 and are bilinear in the fields.

In the case $(d, N_{TT}, N_V, N_H) = (4,2,1,0)$ we can drop the index A labelling the vector multiplets owing to $N_V = 1$ (each vector multiplet can be treated separately because the supersymmetry transformations do not mix the fields of different vector multiplets). The general form of j^μ in $A_\mu j^\mu$ is in this case

$$j^\mu = k_1 \epsilon^{ij} a^i \partial^\mu a^j + k_2 \psi\sigma^\mu\bar\psi + k_3 \chi\sigma^\mu\bar\chi + k_4 \psi\sigma^\mu\bar\chi + \bar k_4 \chi\sigma^\mu\bar\psi$$

where $k_1, k_2, k_3 \in \mathbb{R}$, $k_4 \in \mathbb{C}$ (this j^μ is thus a linear combination of five different real Noether currents). In addition, there are terms of the type met already above, involving one scalar field and two fermions (and no derivative),

$$\phi(k_5\bar\psi\bar\psi + k_6\bar\chi\bar\psi + k_7\bar\chi\bar\chi) + \bar\phi(k_8\bar\psi\bar\psi + k_9\bar\chi\bar\psi + k_{10}\bar\chi\bar\chi) + a^i\bar\lambda^j(k_{11}^{ij}\bar\psi + k_{12}^{ij}\bar\chi) + \text{c.c.}$$

with $k_5, \ldots, k_{12}^{ij} \in \mathbb{C}$. It is easy to verify that all coefficients $k_5, \ldots, k_{12}^{ij}$ must vanish. This comes again from the fact that the $D_\alpha^{(0)i}$-transformations of $\bar\chi$ and $\bar\psi$ contain the H^i_μ; as a consequence, $D_\alpha^{(0)i}(k_5\phi\bar\psi\bar\psi + \ldots)$ contains in particular terms with one scalar field and one H, i.e. terms $\phi H\bar\psi$, $\bar\phi H\bar\psi$, $\phi H\bar\chi$, $\bar\phi H\bar\chi$, $aH\bar\lambda$. Such terms do not occur in $D_\alpha^{(0)i}(A_\mu j^\mu)$. This enforces $k_5 = \ldots = k_{12}^{ij} = 0$. Finally, $D_\alpha^{(0)i}(A_\mu j^\mu) \sim 0$ imposes $k_1 = k_2 = k_3 = k_4 = 0$. This simply follows from the fact that $D_\alpha^{(0)i}(A_\mu j^\mu)$ involves terms with $\bar\lambda$'s owing to $D_\alpha^{(0)i} A_\mu = \epsilon^{ij}(\sigma_\mu\bar\lambda^j)_\alpha$; evidently $\epsilon^{ij}(\sigma_\mu\bar\lambda^j)_\alpha j^\mu \sim 0$ imposes $k_1 = k_2 = k_3 = k_4 = 0$.

The remaining cases $(d, N_{TT}, N_V, N_H) = (4,1,2,0)$ and $(4,1,1,1)$ are similar. Again there terms of the form $A^A_\mu j^\mu_A$ with j's of the form $s\partial s + f\bar f$ and terms $s\bar f\bar f +$ c.c. where s and f stand for scalar fields and fermions respectively. The coefficients of all terms $s\bar f\bar f$ containing $\bar\psi$ or $\bar\chi$ must vanish because of the presence of H in the supersymmetry transformations of $\bar\psi$ or $\bar\chi$. The remaining terms $s\bar f\bar f +$ c.c. have necessarily $s = a^1$ or $s = a^2$ and do not contain fermions of the double tensor multiplet (due to $N_{TT} = 1$); their coefficients must vanish because their supersymmetry transformations contain 3-fermion-terms $(D_\alpha^{(0)i} a^j) ff$ which do not occur in $D_\alpha^{(0)i}(A_\mu j^\mu)$. It is then easy to verify that the terms $A^A_\mu j^\mu_A$ must vanish as well.

Remark. Again, the case $N_{TT}=0$ yields nontrivial on-shell invariants: $(d,N_{TT},N_V,N_H)=(4,0,3,0)$ gives the N=2 supersymmetric extension of the cubic vertices between the vector gauge fields, $(d,N_{TT},N_V,N_H)=(4,0,1,2)$ gives couplings between vector and hypermultiplets of the form $A^A_\mu j^\mu_A+\dots$. These on-shell invariants yield of course the deformations of the free Lagrangian for vector multiplets and hypermultiplets to the standard $N=2$-supersymmetric abelian or nonabelian gauge theories.

4.5 d=5

We shall first discuss the polynomials with $N_\Phi=3$ as they yield nontrivial first order deformations. The cases to be discussed are $(d,N_{TT},N_V,N_H)=(5,3,0,0)$, $(5,1,2,0)$, $(5,1,0,2)$, $(5,2,1,0)$, $(5,2,0,1)$, $(5,1,1,1)$. The first three cases yield the interaction vertices of type A, B and C mentioned in the introduction. It is helpful to observe that, for $d=5$ and $N_\Phi=3$, there are only terms with $(N_\partial,N_f)=(2,0)$ and $(1,2)$, owing to (4.6). Furthermore, there are no nontrivial $(N_\partial,N_f)=(2,0)$-terms involving three scalar fields and no nontrivial $(N_\partial,N_f)=(1,2)$-terms involving a scalar field. Indeed, every Poincaré invariant $(N_\partial,N_f)=(2,0)$-term with three scalar fields s_1,s_2,s_3 can be brought to the form $s_1\partial_\mu s_2\partial^\mu s_3$ by adding trivial terms. However, $s_1\partial_\mu s_2\partial^\mu s_3$ itself is trivial because in the free theory it is on-shell equal to $\frac12\partial_\mu(s_1s_2\partial^\mu s_3-s_3s_2\partial^\mu s_1+s_1s_3\partial^\mu s_2)$. Every Poincaré invariant $(N_\partial,N_f)=(1,2)$-term with a scalar field s is modulo a total derivative a linear combination of terms $sf\partial\bar f$ and $s(\partial f)\bar f$ which vanish on-shell in the free theory. This makes it relatively easy to determine $P_{(d,N_{TT},N_V,N_H)}$ in the various cases. I note that in order to compute $D^{(0)i}_\alpha P_{(d,N_{TT},N_V,N_H)}$, it is often useful to employ

$$D^{(0)i}_\alpha H^j_\mu\approx-\tfrac{\mathrm{i}}{2}\left(\epsilon^{ij}\partial_\mu\chi_\alpha+\delta^{ij}\partial_\mu\psi_\alpha\right)$$

where $\approx$ is again on-shell equality in the free theory.

First order interactions of type A. We start with the case $(d,N_{TT},N_V,N_H)=(5,3,0,0)$ and treat it in some detail to illustrate the calculations. In this case one finds (modulo trivial terms)

$$\begin{aligned}P_{(5,3,0,0)}&=\tfrac12\,k_1^k\epsilon^{ij}H^i_\mu H^j_\nu B^k_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}+k_2^{ijk}H^i_\mu H^{\mu j}a^k+k_3^k\epsilon^{ji}a^i\partial^\mu a^jH^k_\mu\\&\quad+(k_4^i\chi\sigma^\mu\bar\chi+k_5^i\psi\sigma^\mu\bar\psi+k_6^i\psi\sigma^\mu\bar\chi+\bar k_6^i\chi\sigma^\mu\bar\psi)H^i_\mu\end{aligned}$$

where $k_1^i,k_2^{ijk},k_3^i,k_4^i,k_5^i\in\mathbb{R}$, $k_6^i\in\mathbb{C}$ and (without loss of generality) $k_2^{ijk}=k_2^{jik}$. One now computes $D^{(0)i}_\alpha P_{(5,3,0,0)}$. Up to trivial terms the result is a linear combination of terms of the form ψHH, χHH, $\psi H\partial a$, $\chi H\partial a$. There are two types of ψHH-terms: terms $H^j_\mu H^k_\nu\sigma^{\mu\nu}\psi$ which are antisymmetric in jk (one has $H^j_\mu H^k_\nu\sigma^{\mu\nu}\psi=\frac12\epsilon^{jk}\epsilon^{lm}H^l_\mu H^m_\nu\sigma^{\mu\nu}\psi$) and terms $H^j_\mu H^{\mu k}\psi$ which are symmetric in jk. Similarly there are two types of χHH-terms. Vanishing of the coefficients of $H^j_\mu H^k_\nu\sigma^{\mu\nu}\psi$ and $H^j_\mu H^k_\nu\sigma^{\mu\nu}\chi$ imposes

$$\mathrm{i}k_1^i+\epsilon^{ij}k_5^j-k_6^i=0,\quad \mathrm{i}k_1^i+\epsilon^{ij}k_4^j+\bar k_6^i=0.$$

Vanishing of the coefficients of $H^j_\mu H^{\mu k}\psi$ and $H^j_\mu H^{\mu k}\chi$ imposes

$$-\mathrm{i}\epsilon^{ij}k_1^k + \tfrac{1}{2}\,\epsilon^{il}k_2^{jkl} + \delta^{ij}k_5^k + \epsilon^{ij}k_6^k + (j\leftrightarrow k) = 0$$
$$\mathrm{i}\delta^{ij}k_1^k + \tfrac{1}{2}\,k_2^{jki} + \epsilon^{ij}k_4^k + \delta^{ij}\bar{k}_6^k + (j\leftrightarrow k) = 0.$$

Vanishing of the coefficients of $\psi H^k_\mu\partial^\mu a^j$ and $\chi H^k_\mu\partial^\mu a^j$ imposes:

$$-\mathrm{i}k_2^{ikj} - \delta^{ij}k_3^k - 2\mathrm{i}\epsilon^{ij}k_5^k - 2\mathrm{i}\delta^{ij}k_6^k = 0$$
$$-\mathrm{i}\epsilon^{il}k_2^{lkj} + \epsilon^{ij}k_3^k - 2\mathrm{i}\delta^{ij}k_4^k - 2\mathrm{i}\epsilon^{ij}\bar{k}_6^k = 0.$$

These equations give

$$k_2^{ijk} = k_4^i = k_5^i = \mathrm{Re}\,k_6^i = 0,\quad k_3^i = 2k_1^i,\quad \mathrm{Im}\,k_6^i = k_1^i.$$

Choosing k_1^i as deformation parameters, the resulting nontrivial first order deformation is thus

$$L^{(1)}_{(5,3,0,0)} = \tfrac{1}{2}\,k_1^k\epsilon^{ij}H^i_\mu H^j_\nu B^k_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} - 2k_1^i H^i_\mu\epsilon^{jk}a^j\partial^\mu a^k + \mathrm{i}k_1^i H^i_\mu(\psi\sigma^\mu\bar{\chi} - \chi\sigma^\mu\bar{\psi}),$$
$$k_1^i\in\mathbb{R}\ . \tag{4.7}$$

First order interactions of type B. The case $(d,N_{TT},N_V,N_H) = (5,1,2,0)$ is more complex. One has

$$\begin{aligned} P_{(5,1,2,0)} \;=\; & H^i_\mu A^A_\nu(k_1^{iAB}F^{\mu\nu B} + k_2^{iAB}F^B_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}) + a^iF^A_{\mu\nu}(k_3^{iAB}F^{\mu\nu B} + k_4^{iAB}F^B_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}) \\ & +[H^i_\mu\phi^A\partial^\mu(k_5^{iAB}\bar{\phi}^B + k_6^{iAB}\phi^B) + k_7^{ijAkB}H^i_\mu\lambda^{jA}\sigma^\mu\bar{\lambda}^{kB} \\ & +F^A_{\mu\nu}\lambda^{iB}\sigma^{\mu\nu}(k_8^{AiB}\chi + k_9^{AiB}\psi) + \text{c.c.}] \end{aligned}$$

where $k_1^{iAB},\dots,k_4^{iAB}\in\mathbb{R}$, $k_5^{iAB},\dots,k_9^{AiB}\in\mathbb{C}$, and (without loss of generality) $k_2^{iAB} = k_2^{iBA}$, $k_3^{iAB} = k_3^{iBA}$, $k_4^{iAB} = k_4^{iBA}$, $k_5^{iAB} = -\bar{k}_5^{iBA}$, $k_6^{iAB} = -k_6^{iBA}$, $k_7^{ijAkB} = \bar{k}_7^{ikBjA}$ (e.g., $k_6^{iAB} = -k_6^{iBA}$ can be imposed owing to $H^i_\mu\phi^{(A}\partial^\mu\phi^{B)} = \frac{1}{2}\,H^i_\mu\partial^\mu(\phi^A\phi^B)\simeq 0$). $D^{(0)i}_\alpha P_{(5,1,2,0)}\sim 0$ imposes

$$k_2^{iAB} = k_3^{iAB} = k_4^{iAB} = \mathrm{Im}\,k_5^{iAB} = k_6^{iAB} = k_8^{AiB} = k_9^{AiB} = \mathrm{Re}\,k_7^{ijAkB} = 0,$$
$$k_1^{iAB} = -k_1^{iBA} = -2\mathrm{Re}\,k_5^{iAB},\quad \mathrm{Im}\,k_7^{ijAkB} = -k_1^{iAB}\delta^{jk}.$$

This gives

$$L^{(1)}_{(5,1,2,0)} = k_1^{iAB}H^i_\mu(A^A_\nu F^{\mu\nu B} - \tfrac{1}{2}\,\phi^A\partial^\mu\bar{\phi}^B - \tfrac{1}{2}\,\bar{\phi}^A\partial^\mu\phi^B - 2\mathrm{i}\lambda^{jA}\sigma^\mu\bar{\lambda}^{jB}),$$
$$k_1^{iAB} = -k_1^{iBA}\in\mathbb{R}\ . \tag{4.8}$$

First order interactions of type C. The case $(d,N_{TT},N_V,N_H) = (5,1,0,2)$ is quite simple. One has

$$\begin{aligned} P_{(5,1,0,2)} \;=\; & H^i_\mu(k_1^{ijakb}\varphi^{ja}\partial^\mu\varphi^{kb} + k_2^{ijakb}\varphi^{ja}\partial^\mu\bar{\varphi}^{kb} \\ & +k_3^{iab}\rho^a\sigma^\mu\bar{\rho}^b + k_4^{iab}\eta^a\sigma^\mu\bar{\eta}^b + k_5^{iab}\rho^a\sigma^\mu\bar{\eta}^b) + \text{c.c.} \end{aligned}$$

where $k_1^{ijakb},\ldots,k_5^{iab}\in\mathbb{C}$, and, without loss of generality, $k_1^{ijakb}=-k_1^{ikbja}$, $k_2^{ijakb}=-\bar{k}_2^{ikbja}$, $k_3^{iab}=\bar{k}_3^{iba}$, $k_4^{iab}=\bar{k}_4^{iba}$. $D_\alpha^{(0)i}P_{(5,1,0,2)}\sim 0$ imposes in addition $k_3^{iab}=-\bar{k}_4^{iab}$, $k_2^{ijakb}=-2\mathrm{i}\delta^{jk}k_3^{iab}$ and $k_1^{ijakb}=\mathrm{i}\epsilon^{jk}k_5^{iab}$. This yields

$$
\begin{aligned}
L^{(1)}_{(5,1,0,2)} &= k_3^{iab}H^i_\mu(-2\mathrm{i}\varphi^{ja}\partial^\mu\bar{\varphi}^{jb}+\rho^a\sigma^\mu\bar{\rho}^b-\eta^b\sigma^\mu\bar{\eta}^a)\\
&\quad + k_5^{iab}H^i_\mu(\mathrm{i}\epsilon^{jk}\varphi^{ja}\partial^\mu\varphi^{kb}+\rho^a\sigma^\mu\bar{\eta}^b)+\text{c.c.}\ ,\\
&\quad k_3^{iab}=\bar{k}_3^{iba}\in\mathbb{C},\quad k_5^{iab}=k_5^{iba}\in\mathbb{C}.
\end{aligned}
\tag{4.9}
$$

Remaining cases. The remaining cases do not give nontrivial first order deformations and will therefore be discussed only briefly. In the case $(d,N_{TT},N_V,N_H)=(5,2,1,0)$ one has

$$
P_{(5,2,1,0)}=\phi H^i_\mu(k_1^{ij}H^{\mu j}+k_2^{ij}\partial^\mu a^j)+k_3F_{\mu\nu}\chi\sigma^{\mu\nu}\psi+H^i_\mu\lambda^j\sigma^\mu(k_4^{ij}\bar{\psi}+k_5^{ij}\bar{\chi})+\text{c.c.}
$$

where $k_1^{ij},\ldots,k_5^{ij}\in\mathbb{C}$. $D_\alpha^{(0)i}P_{(5,2,1,0)}\sim 0$ gives

$$
k_1^{ij}=\ldots=k_5^{ij}=0.
$$

The case $(d,N_{TT},N_V,N_H)=(5,1,1,1)$ is similar to the case $(5,2,1,0)$, owing to the duality relation between the double tensor multiplet and a hypermultiplet. As a consequence, it does not give nontrivial first order deformations.

In the case $(d,N_{TT},N_V,N_H)=(5,2,0,1)$ one has

$$
\begin{aligned}
P_{(5,2,0,1)} &= H^i_\mu(k_1^{ijk}H^{\mu j}\varphi^k+k_2^{ijk}\varphi^j\partial^\mu a^k+k_3^i\chi\sigma^\mu\bar{\rho}\\
&\quad +k_4^i\psi\sigma^\mu\bar{\rho}+k_5^i\chi\sigma^\mu\bar{\eta}+k_6^i\psi\sigma^\mu\bar{\eta})+\text{c.c.}
\end{aligned}
$$

where $k_1^{ijk},\ldots,k_6^i\in\mathbb{C}$. Again, $D_\alpha^{(0)i}P_{(5,2,0,1)}\sim 0$ imposes

$$
k_1^{ijk}=\ldots=k_6^i=0.
$$

The remaining cases with $d=5$ are those with $N_\Phi=2$ and $N_\Phi=4$. The field polynomials with $N_\Phi=2$ contain terms with $(N_\partial,N_f)=(3,0)$ and $(2,2)$. It is easy to see that all Poincaré invariant terms of these types which are gauge invariant on-shell modulo total derivatives are trivial (an example is $F^{\mu\nu}\partial_\mu H^i_\nu$). Field polynomials with $N_\Phi=4$ contain only terms with $(N_\partial,N_f)=(1,0)$ and $(0,2)$; there are no such Poincaré invariant polynomials with $N_{TT}\geq 1$ which are gauge invariant and N=2 supersymmetric modulo trivial terms.

5 New N=2 supersymmetric gauge theories

5.1 Reformulation of the free double tensor multiplet

The study and construction of the deformations at higher orders is considerably simplified by switching to an alternative (equivalent) formulation of the free double tensor multiplet. The free action (2.1) for the double tensor multiplet is replaced by

$$
S^{(0)}_{TT}=\int d^4x\,(\partial_\mu a^i\partial^\mu a^i+h^i_\mu h^{\mu i}+2h^i_\mu H^{\mu i}-\mathrm{i}\chi\partial\bar{\chi}-\mathrm{i}\psi\partial\bar{\psi})
\tag{5.1}
$$

where the h^i_μ are auxiliary vector fields. Eliminating them by their algebraic equations of motion reproduces (2.1). Since (2.1) and (5.1) agree except for the term $(h^i_\mu + H^i_\mu)(h^{\mu i} + H^{\mu i})$, (5.1) is invariant under the supersymmetry transformations (2.7) supplemented by $D^{(0)i}_\alpha h^j_\mu = -D^{(0)i}_\alpha H^j_\mu$. Furthermore it is convenient to substitute h's for H's in the supersymmetry transformations of the fermions. This is achieved by adding suitable on-shell trivial symmetries to the transformations of the fermions and the h's. The resulting new supersymmetry transformations $\tilde{D}^{(0)i}_\alpha$ are

$$\begin{aligned}
\tilde{D}^{(0)i}_\alpha \bar{\chi}_{\dot\alpha} &= D^{(0)i}_\alpha \bar{\chi}_{\dot\alpha} + \frac{1}{2}\,\sigma^\mu_{\alpha\dot\alpha}\,\epsilon^{ij}\,\frac{\delta L^{(0)}}{\delta h^{\mu j}} \\
\tilde{D}^{(0)i}_\alpha \bar{\psi}_{\dot\alpha} &= D^{(0)i}_\alpha \bar{\psi}_{\dot\alpha} + \frac{1}{2}\,\sigma^\mu_{\alpha\dot\alpha}\,\delta^{ij}\,\frac{\delta L^{(0)}}{\delta h^{\mu j}} \\
\tilde{D}^{(0)i}_\alpha h^{\mu j} &= D^{(0)i}_\alpha h^{\mu j} - \frac{1}{2}\,\sigma^\mu_{\alpha\dot\alpha}\left[\epsilon^{ij}\,\frac{\delta L^{(0)}}{\delta \bar{\chi}_{\dot\alpha}} + \delta^{ij}\,\frac{\delta L^{(0)}}{\delta \bar{\psi}_{\dot\alpha}}\right]
\end{aligned}$$

where $\delta L^{(0)}/\delta\Phi$ is the Euler-Lagrange derivative of the free Lagrangian in (5.1) with respect to Φ. The new transformations differ from the supersymmetry transformations (2.7) only through terms that vanish on-shell. Hence, they fulfill the N=2 supersymmetry algebra (on-shell and modulo gauge transformations). We shall from now on drop the tilde-symbol and denote the new supersymmetry transformations again by $D^{(0)i}_\alpha$,

$$\begin{aligned}
D^{(0)i}_\alpha a^j &= \tfrac{1}{2}\,(\delta^{ij}\chi_\alpha + \epsilon^{ij}\psi_\alpha) \\
D^{(0)i}_\alpha B^j_{\mu\nu} &= \epsilon^{ij}(\sigma_{\mu\nu}\chi)_\alpha + \delta^{ij}(\sigma_{\mu\nu}\psi)_\alpha \\
D^{(0)i}_\alpha h^j_\mu &= \tfrac{\mathrm{i}}{2}\,\partial_\mu(\epsilon^{ij}\chi_\alpha + \delta^{ij}\psi_\alpha) \\
D^{(0)i}_\alpha \chi_\beta &= 0 \\
D^{(0)i}_\alpha \psi_\beta &= 0 \\
D^{(0)i}_\alpha \bar{\chi}_{\dot\alpha} &= -\mathrm{i}\,\partial_{\alpha\dot\alpha}a^i + \epsilon^{ij}h^j_{\alpha\dot\alpha} \\
D^{(0)i}_\alpha \bar{\psi}_{\dot\alpha} &= -\mathrm{i}\,\epsilon^{ij}\partial_{\alpha\dot\alpha}a^j + h^i_{\alpha\dot\alpha}\ .
\end{aligned} \tag{5.2}$$

The gauge transformations $\delta^{(0)}_\varepsilon$ do not change; the auxiliary fields h^i_μ are invariant under these transformations. The free action and supersymmetry transformations of the vector multiplets and hypermultiplets are not changed.

The computations involve the following steps:

1. The auxiliary fields h^i_μ substitute for $(-H^i_\mu)$ in the first order deformations (4.7), (4.8) and (4.9). This is possible because h^i_μ and $-H^i_\mu$ coincide on-shell in the free theory.

2. The second step is the determination of the corresponding first order deformations $\delta^{(1)}_\varepsilon$ and $D^{(1)i}_\alpha$ of the gauge and supersymmetry transformations. That amounts to making the coefficients of the Euler-Lagrange derivatives in (3.7) explicit for the respective first order deformations.

3. One computes $\delta^{(1)}_\varepsilon L^{(1)}$ and $D^{(1)i}_\alpha L^{(1)}$ and seeks an $L^{(2)}$ such that (3.10) and (3.11) hold. If necessary, one then proceeds analogously to higher orders.

5.2 Structure of the supersymmetry deformations

To carry out these computations and to understand the structure of the resulting supersymmetry deformations, the following observation is useful. The first order deformations (4.7), (4.8) and (4.9) have a remarkable property in common: in the fomulation with the auxiliary fields h^i_μ, all of them (and therefore all linear combinations of them too) are of the form

$$L^{(1)} = h^i_\mu j^{\mu i} \tag{5.3}$$

where $j^{\mu i}$ are Noether currents of the free theory,

$$\partial_\mu j^{\mu i} = \sum_\Phi (\Delta^i \Phi) \frac{\delta L^{(0)}}{\delta \Phi} \ . \tag{5.4}$$

Here Δ^i generates the global symmetry of the free Lagrangian corresponding by Noether's first theorem to $j^{\mu i}$. Owing to the supersymmetry transformation of h^i_μ in (5.2), one has

$$D^{(0)i}_\alpha L^{(1)} = \tfrac{\mathrm{i}}{2}\, \partial_\mu(\epsilon^{ij}\chi_\alpha + \delta^{ij}\psi_\alpha) j^{\mu j} + h^j_\mu\, D^{(0)i}_\alpha j^{\mu j}. \tag{5.5}$$

Owing to (5.4), the first term on the right hand side is a linear combination of the Euler-Lagrange derivatives of $L^{(0)}$ modulo a total derivative,

$$\tfrac{\mathrm{i}}{2}\, \partial_\mu(\epsilon^{ij}\chi_\alpha + \delta^{ij}\psi_\alpha) j^{\mu j} \simeq -\sum_\Phi \tfrac{\mathrm{i}}{2}\, (\epsilon^{ij}\chi_\alpha + \delta^{ij}\psi_\alpha)(\Delta^j \Phi) \frac{\delta L^{(0)}}{\delta \Phi} \ . \tag{5.6}$$

Hence, in order that a term of the form (5.3) gives a supersymmetric first order deformation, the second term on the right hand side of (5.5) must also be a linear combination of the Euler-Lagrange derivatives of $L^{(0)}$ (modulo a total derivative),

$$h^j_\mu\, D^{(0)i}_\alpha j^{\mu j} \simeq -\sum_\Phi (\Delta^i_\alpha \Phi) \frac{\delta L^{(0)}}{\delta \Phi} \ , \tag{5.7}$$

for some transformations Δ^i_α. Equations (5.5) through (5.7) yield then first order deformations of the supersymmetry transformations of the following form:

$$D^{(1)i}_\alpha \Phi = \Delta^i_\alpha \Phi + \tfrac{\mathrm{i}}{2}\, (\epsilon^{ij}\chi_\alpha + \delta^{ij}\psi_\alpha)\Delta^j \Phi \ . \tag{5.8}$$

Note that (5.6) holds for any Noether current $j^{\mu i}$, in contrast to (5.7). The currents which appear in (4.7), (4.8) and (4.9) have thus the special property to fulfill (5.7).

Even though the supersymmetry transformations do get nontrivially deformed, one would not expect that the supersymmetry algebra gets deformed (otherwise the deformed supersymmetry algebra would contain global symmetries generated by transformations which are at least quadratic in the fields – a very unlikely possibility). Indeed one finds in all examples to be discussed in the following that the supersymmetry algebra in the deformed model is the standard one, i.e., it has the form

$$[\delta_\xi, \delta_{\xi'}] \approx \xi^\mu \partial_\mu + \delta_{\mathrm{gauge}} \tag{5.9}$$

where $\approx$ is on-shell-equality in the deformed model (note: in the previous sections $\approx$ denoted on-shell-equality in the free model!), δ_ξ are the deformed supersymmetry transformations, $\xi^\mu \partial_\mu$ is an ordinary translation with parameter ξ^μ as in (2.13), and δ_{gauge} is a deformed gauge transformation.

5.3 First example (type C)

The simplest examples of the first order deformations (4.9) are those involving only one hypermultiplet. In these examples we can thus drop the index a distinguishing different hypermultiplets. Furthermore we choose $k_3^i = 0$ and $k_5^i = \mathrm{i}g^i$ with real g^i. In the formulation with the auxiliary fields h_μ^i, (4.9) then becomes

$$L^{(1)} = g^i h_\mu^i(\epsilon^{jk}\varphi^j\partial^\mu\varphi^k - \mathrm{i}\rho\sigma^\mu\bar{\eta} + \text{c.c.}), \quad g^i \in \mathbb{R} \ .$$

Sketch of the computation. $L^{(1)}$ is indeed of the form (5.3), with $j^{\mu i} = g^i j^\mu$ and j^μ the term in parenthesis. We have $\delta_\varepsilon^{(0)} L^{(1)} = 0$, i.e., the gauge transformations do not get deformed at first order. In order to determine the first order deformations of the supersymmetry transformations one computes $D_\alpha^{(0)i}L^{(1)}$. The result is

$$\begin{aligned} D_\alpha^{(0)i}L^{(1)} \simeq \Gamma_\alpha^i \Big[&\epsilon^{jk}\varphi^j\,\frac{\delta L^{(0)}}{\delta\bar{\varphi}^k} + \epsilon^{jk}\bar{\varphi}^j\,\frac{\delta L^{(0)}}{\delta\varphi^k} + \eta_\beta\,\frac{\delta L^{(0)}}{\delta\rho_\beta} - \rho_\beta\,\frac{\delta L^{(0)}}{\delta\eta_\beta} + \bar{\eta}_{\dot{\alpha}}\,\frac{\delta L^{(0)}}{\delta\bar{\rho}_{\dot{\alpha}}} - \bar{\rho}_{\dot{\alpha}}\,\frac{\delta L^{(0)}}{\delta\bar{\eta}_{\dot{\alpha}}}\Big] \\ &+\mathrm{i}g^j h_{\alpha\dot{\alpha}}^j\Big[\varphi^i\,\frac{\delta L^{(0)}}{\delta\bar{\rho}_{\dot{\alpha}}} - \epsilon^{ik}\bar{\varphi}^k\,\frac{\delta L^{(0)}}{\delta\bar{\eta}_{\dot{\alpha}}}\Big] + \mathrm{i}g^k(\varphi^i\sigma_{\mu\nu}\rho - \epsilon^{ij}\bar{\varphi}^j\sigma_{\mu\nu}\eta)_\alpha\,\frac{\delta L^{(0)}}{\delta B_{\mu\nu}^k} \end{aligned}$$

where

$$\Gamma_\alpha^i = \tfrac{\mathrm{i}}{2}\, g^j(\epsilon^{ij}\chi_\alpha + \delta^{ij}\psi_\alpha). \tag{5.10}$$

Owing to $j^{\mu i} = g^i j^\mu$, we have $\Delta^i = g^i\Delta$, and (5.8) reads

$$D_\alpha^{(1)i}\Phi = \Delta_\alpha^i\Phi + \Gamma_\alpha^i\Delta\Phi$$

where Δ_α^i acts nontrivially only on $\bar{\rho}$, $\bar{\eta}$ and the B's,

$$\begin{aligned} &\Delta_\alpha^i\bar{\rho}_{\dot{\alpha}} = -\mathrm{i}g^k h_{\alpha\dot{\alpha}}^k\,\varphi^i \\ &\Delta_\alpha^i\bar{\eta}_{\dot{\alpha}} = \mathrm{i}g^k h_{\alpha\dot{\alpha}}^k\,\epsilon^{ij}\bar{\varphi}^j \\ &\Delta_\alpha^i B_{\mu\nu}^j = \mathrm{i}\,g^j(\epsilon^{ik}\bar{\varphi}^k\sigma_{\mu\nu}\eta - \varphi^i\sigma_{\mu\nu}\rho)_\alpha \\ &\Delta_\alpha^i(\text{other fields}) = 0, \end{aligned}$$

and Δ rotates the fields of the hypermultiplet,

$$\begin{gathered} \Delta\varphi^i = \epsilon^{ij}\bar{\varphi}^j, \quad \Delta\bar{\varphi}^i = \epsilon^{ij}\varphi^j \\ \Delta\rho_\alpha = -\eta_\alpha\ , \quad \Delta\eta_\alpha = \rho_\alpha\ , \quad \Delta\bar{\rho}_{\dot{\alpha}} = -\bar{\eta}_{\dot{\alpha}}\ , \quad \Delta\bar{\eta}_{\dot{\alpha}} = \bar{\rho}_{\dot{\alpha}} \\ \Delta(\text{other fields}) = 0. \end{gathered} \tag{5.11}$$

Next one computes $D_\alpha^{(1)i}L^{(1)}$. One easily verifies $\Delta L^{(1)} = 0$. Using in addition $D_\alpha^{(1)i}h_\mu^j = 0$ and $D_\alpha^{(0)i}(g^j h_\mu^j) = \partial_\mu\Gamma_\alpha^i$, it is straightforward to verify that one gets

$$D_\alpha^{(1)i}L^{(1)} = -D_\alpha^{(0)i}(g^j h^{\mu j} g^k h_\mu^k\varphi^l\bar{\varphi}^l).$$

Result. This completes the construction of the deformation. Indeed, since the previous equation does not involve the free field equations, we can set $D_\alpha^{(2)i} = 0$ and the term in parenthesis on the right hand side can be taken as $L^{(2)}$: we have $D_\alpha^{(1)i} L^{(2)} = 0$ and $\delta_\varepsilon^{(0)} L^{(2)} = 0$. Hence, one gets a deformed Lagrangian $L = L^{(0)} + L^{(1)} + L^{(2)}$ which is invariant under the gauge transformations $\delta_\varepsilon^{(0)}$ and the supersymmetry transformations $D_\alpha^i = D_\alpha^{(0)i} + D_\alpha^{(1)i}$. The result can be written more compactly in terms of an auxiliary covariant derivative

$$\hat{D}_\mu = \partial_\mu - g^i h_\mu^i \Delta \tag{5.12}$$

with Δ as in (5.11). The deformed Lagrangian for the double tensor multiplet and the hyper multiplet reads then

$$L = L_{TT}^{(0)} + \hat{D}_\mu \varphi^i \hat{D}^\mu \bar{\varphi}^i - \mathrm{i}\rho\sigma^\mu \hat{D}_\mu \bar{\rho} - \mathrm{i}\eta\sigma^\mu \hat{D}_\mu \bar{\eta} \tag{5.13}$$

with $L_{TT}^{(0)}$ as in (5.1). The deformed supersymmetry transformations are

$$\begin{aligned}
D_\alpha^i \varphi^j &= \epsilon^{ij}\rho_\alpha + \Gamma_\alpha^i \epsilon^{jk} \bar{\varphi}^k \\
D_\alpha^i \bar{\varphi}^j &= \delta^{ij}\eta_\alpha + \Gamma_\alpha^i \epsilon^{jk} \varphi^k \\
D_\alpha^i \rho_\beta &= -\Gamma_\alpha^i \eta_\beta \\
D_\alpha^i \eta_\beta &= \Gamma_\alpha^i \rho_\beta \\
D_\alpha^i \bar{\rho}_{\dot\alpha} &= -\mathrm{i}\,\epsilon^{ij} \hat{D}_{\alpha\dot\alpha} \bar{\varphi}^j - \Gamma_\alpha^i \bar{\eta}_{\dot\alpha} \\
D_\alpha^i \bar{\eta}_{\dot\alpha} &= -\mathrm{i}\,\hat{D}_{\alpha\dot\alpha} \varphi^i + \Gamma_\alpha^i \bar{\rho}_{\dot\alpha} \\
D_\alpha^i B_{\mu\nu}^j &= (\epsilon^{ij}\sigma_{\mu\nu}\chi + \delta^{ij}\sigma_{\mu\nu}\psi + \mathrm{i}g^j \epsilon^{ik}\bar{\varphi}^k \sigma_{\mu\nu}\eta - \mathrm{i}g^j \varphi^i \sigma_{\mu\nu}\rho)_\alpha \\
D_\alpha^i &= D_\alpha^{(0)i} \text{ on the other fields.}
\end{aligned} \tag{5.14}$$

It can now be explicitly verified that (5.9) holds in the deformed model.

One may finally eliminate the auxiliary fields h_μ^i by their algebraic equations of motion. That amounts to the identification (both in the action and symmetry transformations)

$$h_\mu^i \equiv -\tfrac{1}{2}\, K^{ij}(H_\mu^j + g^j \epsilon^{kl}\varphi^k \partial_\mu \varphi^l - \mathrm{i}g^j \rho\sigma_\mu\bar{\eta} + \text{c.c.}), \quad K^{ij} = \delta^{ij} - \frac{g^i g^j \varphi^k \bar{\varphi}^k}{1 + g^m g^m \varphi^n \bar{\varphi}^n}\ . \tag{5.15}$$

5.4 Second example (type B)

The simplest first order deformation (4.8) arises when only two vector multiplets are involved. So, in the following we take $A = 1, 2$. This gives $k^{iAB} = -g^i \epsilon^{AB}$ in (4.8) and

$$L^{(1)} = g^i h_\mu^i \epsilon^{AB}(A_\nu^A F^{\mu\nu B} - \tfrac{1}{2}\,\phi^A \partial^\mu \bar{\phi}^B - \tfrac{1}{2}\,\bar{\phi}^A \partial^\mu \phi^B - 2\mathrm{i}\lambda^{jA}\sigma^\mu \bar{\lambda}^{jB}), \quad g^i \in \mathbb{R}\ .$$

Sketch of the computation. Again $L^{(1)}$ has the form (5.3), with $j^{\mu i} = g^i j^\mu$. This time $\delta_\varepsilon^{(0)} L^{(1)}$ does not vanish (even modulo a total derivative), i.e., the gauge transformations are deformed. One has

$$\delta_\varepsilon^{(0)} L^{(1)} \simeq -\tfrac{1}{4}\, g^i \epsilon^{AB} \varepsilon^A \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma B}\, \frac{\delta L^{(0)}}{\delta B_{\mu\nu}^i} + g^i h_\mu^i \epsilon^{AB} \varepsilon^A\, \frac{\delta L^{(0)}}{\delta A_\mu^B}\ .$$

The first order deformation of the gauge transformations is therefore

$$\begin{aligned}\delta_\varepsilon^{(1)} A_\mu^A &= g^i h_\mu^i \epsilon^{AB}\varepsilon^B\\ \delta_\varepsilon^{(1)} B_{\mu\nu}^i &= \tfrac{1}{4}\, g^i \varepsilon^A \epsilon^{AB}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma B}, \quad \delta_\varepsilon^{(1)}(\text{other fields}) = 0\end{aligned}$$

Next one computes $D_\alpha^{(0)i}L^{(1)}$. The result is again of the form

$$D_\alpha^{(0)i}L^{(1)} \simeq -\sum_\Phi (D_\alpha^{(1)i}\Phi)\,\frac{\delta L^{(0)}}{\delta\Phi}\ , \quad D_\alpha^{(1)i}\Phi = \Delta_\alpha^i\Phi + \Gamma_\alpha^i\Delta\Phi$$

with Γ_α^i as in (5.10), but now Δ_α^i is given by

$$\begin{aligned}&\Delta_\alpha^i\lambda_\beta^{jA} = -\mathrm{i}\epsilon^{ij}\epsilon^{AB}g^k h_\mu^k\, A_\nu^B\,\sigma^{\mu\nu}{}_{\alpha\beta}\\ &\Delta_\alpha^i\bar\lambda_{\dot\alpha}^{jA} = -\tfrac{\mathrm{i}}{2}\,\delta^{ij}\epsilon^{AB}g^k h_{\alpha\dot\alpha}^k\,\bar\phi^B\\ &\Delta_\alpha^i B_{\mu\nu}^j = \mathrm{i}g^j\epsilon^{AB}(\bar\phi^A\sigma_{\mu\nu}\lambda^{iB} + \epsilon^{ik}A_{[\mu}^A\sigma_{\nu]}\bar\lambda^{kB})_\alpha\\ &\Delta_\alpha^i(\text{other fields}) = 0\end{aligned}$$

and Δ rotates the fields of the vector multiplets,

$$\Delta X^A = -\epsilon^{AB}X^B \quad \text{for} \quad X^A \in \{A_\mu^A, \phi^A, \bar\phi^A, \lambda_\alpha^{Ai}, \bar\lambda_{\dot\alpha}^{iA}\}, \quad \Delta(\text{other fields}) = 0. \tag{5.16}$$

To determine the second order deformation, one must compute both $\delta_\varepsilon^{(1)}L^{(1)}$ and $D_\alpha^{(1)i}L^{(1)}$. As in the first example, $L^{(1)}$ is Δ-invariant which makes it to easy to compute $D_\alpha^{(1)i}L^{(1)}$. The result is

$$\begin{aligned}\delta_\varepsilon^{(1)}L^{(1)} &= -\delta_\varepsilon^{(0)}L^{(2)} + \tfrac{1}{2}\, g^i g^j h^{\rho j}A^{\sigma A}\varepsilon^A\epsilon_{\mu\nu\rho\sigma}\,\frac{\delta L^{(0)}}{\delta B_{\mu\nu}^i}\\ D_\alpha^{(1)i}L^{(1)} &= -D_\alpha^{(0)i}L^{(2)}\end{aligned}$$

where

$$L^{(2)} = \tfrac{1}{2}\,(g^i h^{\mu i}g^j h^{\nu j}A_\mu^A A_\nu^A - g^i h^{\mu i}g^j h_\mu^j A^{\nu A}A_\nu^A + g^i h^{\mu i}g^j h_\mu^j\phi^A\bar\phi^A).$$

The second order deformations of the gauge and supersymmetry transformations are thus

$$\delta_\varepsilon^{(2)}B_{\mu\nu}^i = -\tfrac{1}{2}\, g^i\varepsilon^A\epsilon_{\mu\nu\rho\sigma}g^j h^{\rho j}A^{\sigma A}, \quad \delta_\varepsilon^{(2)}(\text{other fields}) = 0, \quad D_\alpha^{(2)i} = 0.$$

Result. This completes the construction of the deformation. Indeed, one has $D_\alpha^{(1)i}L^{(2)} = 0$ and $\delta_\varepsilon^{(2)}L^{(2)} = 0$. Hence, $L = L^{(0)} + L^{(1)} + L^{(2)}$ is invariant (modulo total derivatives) under $D_\alpha^i = D_\alpha^{(0)i} + D_\alpha^{(1)i}$ and $\delta_\varepsilon = \delta_\varepsilon^{(0)} + \delta_\varepsilon^{(1)} + \delta_\varepsilon^{(2)}$. Again, the deformed Lagrangian and symmetry transformations can be written more compactly in terms of auxiliary covariant derivatives (5.12) where now Δ is given by (5.16). The deformed Lagrangian for two vector multiplets and the double tensor multiplets reads then

$$L = L_{TT}^{(0)} - \tfrac{1}{4}\,\hat F_{\mu\nu}^A\hat F^{\mu\nu A} + \tfrac{1}{2}\,\hat D_\mu\phi^A\hat D^\mu\bar\phi^A - 2\mathrm{i}\lambda^{iA}\sigma^\mu\hat D_\mu\bar\lambda^{iA} \tag{5.17}$$

with $L^{(0)}_{TT}$ as in (5.1) and

$$\hat{F}^A_{\mu\nu} = \hat{D}_\mu A^A_\nu - \hat{D}_\nu A^A_\mu = F^A_{\mu\nu} + g^i h^i_\mu \epsilon^{AB} A^B_\nu - g^i h^i_\nu \epsilon^{AB} A^B_\mu \ . \tag{5.18}$$

The deformed gauge transformations are

$$\begin{aligned}
&\delta_\varepsilon A^A_\mu = \partial_\mu \varepsilon^A + g^i h^i_\mu \epsilon^{AB} \varepsilon^B =: \hat{D}_\mu \varepsilon^A \\
&\delta_\varepsilon B^i_{\mu\nu} = \tfrac{1}{4}\, g^i \varepsilon^A \epsilon^{AB} \epsilon_{\mu\nu\rho\sigma} \hat{F}^{\rho\sigma B} + \partial_\mu \varepsilon^i_\nu - \partial_\nu \varepsilon^i_\mu \\
&\delta_\varepsilon (\text{other fields}) = 0
\end{aligned} \tag{5.19}$$

and the deformed supersymmetry transformations are

$$\begin{aligned}
D_\alpha{}^i A^A_\mu &= \epsilon^{ij} (\sigma_\mu \bar{\lambda}^{jA})_\alpha - \epsilon^{AB} \Gamma^i_\alpha A^B_\mu \\
D_\alpha{}^i \phi^A &= 2\lambda^{iA}_\alpha - \epsilon^{AB} \Gamma^i_\alpha \phi^B \\
D_\alpha{}^i \bar{\phi}^A &= -\epsilon^{AB} \Gamma^i_\alpha \bar{\phi}^B \\
D_\alpha{}^i \lambda^{jA}_\beta &= -\tfrac{\mathrm{i}}{2}\, \epsilon^{ij} \sigma^{\mu\nu}_{\alpha\beta} \hat{F}^A_{\mu\nu} - \epsilon^{AB} \Gamma^i_\alpha \lambda^{jB}_\beta \\
D_\alpha{}^i \bar{\lambda}^{jA}_{\dot\alpha} &= -\tfrac{\mathrm{i}}{2}\, \delta^{ij}\, \hat{D}_{\alpha\dot\alpha} \bar{\phi}^A - \epsilon^{AB} \Gamma^i_\alpha \bar{\lambda}^{jB}_{\dot\alpha} \\
D_\alpha{}^i B^j_{\mu\nu} &= (\epsilon^{ij} \sigma_{\mu\nu} \chi + \delta^{ij} \sigma_{\mu\nu} \psi + \mathrm{i} g^j \epsilon^{AB} \bar{\phi}^A \sigma_{\mu\nu} \lambda^{iB} + \mathrm{i} g^j \epsilon^{AB} \epsilon^{ik} A^A_{[\mu} \sigma_{\nu]} \bar{\lambda}^{kB})_\alpha \\
D^i_\alpha &= D^{(0)i}_\alpha \text{ on the other fields.}
\end{aligned} \tag{5.20}$$

Again, one may check that (5.9) holds. Elimination of the auxiliary fields h^i_μ is more cumbersome than in the first model but it is clearly possible.

5.5 Third example (type A)

In the formulation with the auxiliary fields, (4.7) becomes

$$L^{(1)} = \tfrac{1}{2}\, g^k \epsilon^{ij} h^i_\mu h^j_\nu B^k_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} + 2 g^i h^i_\mu \epsilon^{jk} a^j \partial^\mu a^k + \mathrm{i} g^i h^i_\mu (\chi \sigma^\mu \bar{\psi} - \psi \sigma^\mu \bar{\chi}), \quad g^i \in \mathbb{R} \tag{5.21}$$

where we have renamed k^i_1 to g^i. This is again of the form (5.3), with

$$j^{\mu i} = g^i j^\mu, \quad j^\mu = \epsilon^{kj} h^j_\nu B^k_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} + 2\epsilon^{jk} a^j \partial^\mu a^k + \mathrm{i} \chi \sigma^\mu \bar{\psi} - \mathrm{i} \psi \sigma^\mu \bar{\chi} \ .$$

This time one gets

$$\delta^{(0)}_\varepsilon L^{(1)} \simeq -\sum_\Phi (\delta^{(1)}_\varepsilon \Phi) \, \frac{\delta L^{(0)}}{\delta \Phi} \ , \quad D^{(0)i}_\alpha L^{(1)} \simeq -\sum_\Phi (D^{(1)i}_\alpha \Phi) \, \frac{\delta L^{(0)}}{\delta \Phi}$$

where

$$\delta^{(1)}_\varepsilon B^i_{\mu\nu} = \epsilon^{ij} g^k (h^j_\mu \varepsilon^k_\nu - h^j_\nu \varepsilon^k_\mu), \quad \delta^{(1)}_\varepsilon (\text{other fields}) = 0 \tag{5.22}$$

$$\begin{aligned}
D^{(1)i}_\alpha a^j &= \Gamma^i_\alpha \epsilon^{jk} a^k \\
D^{(1)i}_\alpha B^j_{\mu\nu} &= \mathrm{i}[g^j \sigma_{\mu\nu} (\epsilon^{ik} \chi - \delta^{ik} \psi) a^k + \tfrac{1}{2}\, g^k (\epsilon^{ij} \psi - \delta^{ij} \chi) B^k_{\mu\nu}]_\alpha \\
D^{(1)i}_\alpha h^j_\mu &= \tfrac{\mathrm{i}}{2}\, g^j (\delta^{ik} \chi - \epsilon^{ik} \psi) h^k_\mu \\
D^{(1)i}_\alpha \chi_\beta &= \Gamma^i_\alpha \psi_\beta \\
D^{(1)i}_\alpha \psi_\beta &= -\Gamma^i_\alpha \chi_\beta \\
D^{(1)i}_\alpha \bar{\chi}_{\dot\alpha} &= \mathrm{i} g^j h^j_{\alpha\dot\alpha} \epsilon^{ik} a^k + \Gamma^i_\alpha \bar{\psi}_{\dot\alpha} \\
D^{(1)i}_\alpha \bar{\psi}_{\dot\alpha} &= -\mathrm{i} g^j h^j_{\alpha\dot\alpha} a^i - \Gamma^i_\alpha \bar{\chi}_{\dot\alpha}
\end{aligned} \tag{5.23}$$

with Γ^i_α as in (5.10). In contrast to the first two examples, $D^{(1)i}_\alpha h^i_\mu$ does not vanish. This makes this example more complicated. To determine $L^{(2)}$, one must compute $\delta^{(1)}_\varepsilon L^{(1)}$ and $D^{(1)i}_\alpha L^{(1)}$. The results are

$$\delta^{(1)}_\varepsilon L^{(1)} \simeq 0, \quad D^{(1)i}_\alpha L^{(1)} \sim -D^{(0)i}_\alpha L^{(2)} \tag{5.24}$$

where

$$\begin{aligned} L^{(2)} &= g^i g^j h^{\mu i} h^j_\mu a^k a^k - g^i g^i a^j h^{\mu j} a^k h^k_\mu + \tfrac{1}{3}\, g^i g^i \epsilon^{jk} a^j \partial_\mu a^k \epsilon^{lm} a^l \partial^\mu a^m \\ &\quad - g^i g^i a^j h^j_\mu (\psi\sigma^\mu\bar{\chi} + \chi\sigma^\mu\bar{\psi}) + \tfrac{\mathrm{i}}{2}\, g^i g^i \epsilon^{jk} a^j \partial_\mu a^k (\chi\sigma^\mu\bar{\psi} - \psi\sigma^\mu\bar{\chi}) \\ &\quad - \tfrac{1}{4}\, g^i g^i (\psi\psi\bar{\chi}\bar{\chi} + \chi\chi\bar{\psi}\bar{\psi}). \end{aligned} \tag{5.25}$$

Note that $L^{(2)}$ is invariant under $\delta^{(0)}_\varepsilon$. Hence the two equations (5.24) are compatible. (5.24) shows also that there is no second order deformation of the gauge transformations. However, the second order deformation of the supersymmetry transformations does not vanish in this case because the free field equations are used in the second equation (5.24) (recall that $\sim$ stands for on-shell equality in the free theory modulo total derivatives). To go on along the above lines, one must determine $D^{(2)i}_\alpha$, compute $D^{(2)i}_\alpha L^{(1)} + D^{(1)i}_\alpha L^{(2)}$ etc.

Of course, it is at this stage not completely clear whether the deformation exists at all orders ≥ 3. However, there are good reasons to assume that it does exist. A simple inductive argument shows that all $L^{(r)}$ which one would get by continuing the above procedure have dimension 4 when one assigns dimension -1 to the coupling constants g^i. Hence, $L^{(r)}$ would be a linear combination of field monomials $M^{(r)}$ of dimension $r+4$, with coefficients of order r in the g^i. Furthermore $M^{(r)}$ would have degree $r+2$ in the fields. Note that $L^{(2)}$ involves only the fields $a^i, h^i_\mu, \psi, \chi, \bar{\psi}, \bar{\chi}$ (but not the $B^i_{\mu\nu}$). Assume now that $L^{(3)}$, $L^{(4)}$, ...can be chosen such that they involve also only these fields. The field monomials $M^{(r)}$, $r \geq 2$ would then fulfill

$$r \geq 2: \quad N_h + N_a + N_f = r+2, \quad N_\partial + 2N_h + N_a + \tfrac{3}{2}\, N_f = r+4$$

where N_h, N_a, N_f are the degrees in the h^i_μ, a^i and the fermions respectively, and N_∂ is the number of derivatives. This implies

$$r \geq 2: \quad N_\partial + N_h + \tfrac{1}{2}\, N_f = 2.$$

Hence, each $M^{(k)}$ could only have $(N_\partial, N_h, N_f, N_a) = (2,0,0,k+2)$, $(1,1,0,k+1)$, $(1,0,2,k)$, $(0,2,0,k)$, $(0,1,2,k-1)$ or $(0,0,4,k-2)$. Modulo trivial terms, this would yield

$$\begin{aligned} r \geq 2: \quad L^{(r)} &= A^{(r)ij}(g,a)\partial_\mu a^i \partial^\mu a^j + B^{(r)ij}(g,a) h^i_\mu \partial^\mu a^j + C^{(r)ij}(g,a) h^i_\mu h^{\mu j} \\ &\quad + D^{(r)ijk}(g,a) f^i \sigma^\mu \bar{f}^j \partial_\mu a^k + E^{(r)ijk}(g,a) f^i \sigma^\mu \bar{f}^j h^k_\mu \\ &\quad + F^{(r)ijkl}(g,a) f^i f^j \bar{f}^k \bar{f}^l \end{aligned} \tag{5.26}$$

where $\{f^i\} = \{\psi, \chi\}$ and $A^{(r)ij}(g,a), \ldots, F^{(r)ijkl}(g,a)$ are polynomials in the g^i and the undifferentiated a^i. In particular the h^i_μ could be eliminated algebraically also in the

deformed theory and the complete deformation of the gauge transformations would be given by $\delta_\varepsilon = \delta_\varepsilon^{(0)} + \delta_\varepsilon^{(1)}$. The supersymmetry transformations may of course receive higher order contributions $D_\alpha^{(r)i}$. One can check that the supersymmetry algebra takes to first order again the standard form (5.9).

Remark. A sufficient condition for the existence of $L^{(r)}$ as in (5.26) to all orders can be formulated in cohomological terms. As remarked at the end of section 3, the deformation problem can be reformulated as a cohomological problem for an extended BRST differential $s^{(0)}$ which encodes the zeroth order gauge and supersymmetry transformations. It is then easy to show that the existence of $L^{(r)}$ as in (5.26) is controlled by the on-shell cohomology of $s^{(0)}$ (modulo total derivatives) in a space of Poincaré invariant local field polynomials in the $a^i, h^i_\mu, \psi, \chi, \bar{\psi}, \bar{\chi}$ and their derivatives which depend in addition linearly on the supersymmetry ghosts (as it is the cohomology at ghost number 1 which enters here; the ghosts of the gauge transformations do not come into play here because $\delta_\varepsilon^{(0)}$ and $\delta_\varepsilon^{(1)}$ act nontrivially only on the $B^i_{\mu\nu}$). Vanishing of that cohomology would guarantee the existence of $L^{(r)}$ as in (5.26) for all r. It is in fact reasonable to believe that this cohomology indeed vanishes because its counterpart for N=1 supersymmetric models with linear multiplets vanishes (this can be shown as in [10], owing to the fact that free linear multiplets have "QDS-structure" on-shell).

6 Comment on an N=1 superfield construction

It has been already mentioned that the free N=2 double tensor multiplet consists of two free N=1 linear multiplets. Similarly, a free N=2 vector multiplet consists of one free N=1 vector multiplet and one free N=1 chiral multiplet, while a free N=2 hypermultiplet consists of two N=1 chiral multiplets. It is well-known that all these N=1 multiplets have off-shell superfield descriptions involving auxiliary fields. These superfields might also be used to construct N=2 supersymmetric interactions involving the double tensor multiplet. A promising step into that direction is the construction in [6]. It provides N=1 supersymmetric interactions of the sought type between N=1 linear multiplets and N=1 vector multiplets, and it can be extended so as to include N=1 chiral multiplets. Some of the resulting models will even be N=2 supersymmetric – an example has been recently given in [5]. However, it is far from obvious how one can sieve out the models with a second supersymmetry systematically.

Acknowledgements. The author acknowledges discussions with Sergei Kuzenko and Ulrich Theis and was supported by a DFG habilitation grant.

References

[1] J. Louis and K. Förger, Nucl. Phys. Proc. Suppl. 55B (1997) 33 (hep-th/9611184)

[2] D. Freedman and P.K. Townsend, Nucl. Phys. B 177 (1981) 282

[3] M. Henneaux and B. Knaepen, Phys. Rev. D 56 (1997) 6076 (hep-th/9706119)

[4] P. Claus, B. de Wit, M. Faux, B. Kleijn, R. Siebelink and P. Termonia, Phys. Lett. B 373 (1996) 81 (hep-th/9512143);
P. Claus, B. de Wit, M. Faux and P. Termonia, Nucl. Phys. B 491 (1997) 201 (hep-th/9612203);
N. Dragon and U. Theis, Phys. Lett. B 446 (1999) 314 (hep-th/9805199);
N. Dragon, E. Ivanov, S. Kuzenko, E. Sokatchev and U. Theis, Nucl. Phys. B 538 (1999) 411 (hep-th/9805152)

[5] U. Theis, Phys. Lett. B, to appear (hep-th/0005044)

[6] F. Brandt and U. Theis, Nucl. Phys. B 550 (1999) 495 (hep-th/9811180)

[7] J. Wess and J. Bagger, *Supersymmetry and supergravity*. Princeton Series in Physics (Princeton Univ. Press, Princeton, 1992)

[8] F. Brandt, M. Henneaux and A. Wilch, Nucl. Phys. B 510 (1998) 640 (hep-th/9705007);
F. Brandt, J. Math. Phys. 40 (1999) 1023 (hep-th/9804153)

[9] G. Barnich and M. Henneaux, Phys. Lett. B 311 (1993) 123 (hep-th/9304057)

[10] F. Brandt, Nucl. Phys. B 392 (1993) 428;
F. Brandt, Ann. Phys. (NY) 259 (1997) 253 (hep-th/9609192)